



# Construction of a geometric reference model for automatic non-ideality evaluation of an acquired high-density workpiece

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## Article Information

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## Abstract

### Purpose:

For some years now, our research group has been developing a new methodology for automatic tolerance inspection starting from an acquired high-density 3D model. In this paper, with a view to grouping together all the information recognisable in a scanned object, a new data structure, called Recognised Geometric Model (RGM), is proposed. Based on this data structure, the evaluation of the non-idealities of the acquired object (form, orientation and location non-idealities) can be automatically carried out.

### Method:

RGM is the result of an approach founded on the concepts of non-ideal feature and intrinsic nominal reference. The object to be inspected is segmented into a set of non-ideal features and, for each of them, one or more intrinsic nominal references are identified. An Intrinsic Nominal Reference is detected when a geometric property has been recognised to be common to a set of adjacent points in the 3D data set representing the acquired object. The recognition of these references from a scanned object is carried out based on some rules which, therefore, play a leading role in the definition of the domain of the representable entities within RGM.

### Result:

New and old categories of form non-idealities are here defined and some procedures are proposed for a more robust process of verification of traditional tolerance categories (such as the straightness of a cylinder generatrix).

### Discussion & Conclusion:

When using the RGM, tolerances can be specified according to the set of available and recognisable intrinsic nominal references. This allows for the automatic geometric inspection of the workpiece. However, the approach here proposed does not rule out the possibility of querying the RGM data structure by explicit geometric product specifications, in order to gather some quantitative information concerning special intrinsic geometric parameters and/or non-idealities.

## 1 Introduction

For the last few years the development of reliable, effective, and automated geometric inspection systems has been arousing great interest. This is due, on the one hand, to the increase in geometric complexity and product variety, and, on the other hand, to the ever higher demands for geometric accuracy.

The design geometric requirements can be expressed by using the geometric product specifications defined by the International Organization for Standardization (ISO) or by the ANSI/ASME. Some years ago, the ISO proposed a new language, usually acronymed GPS (Geometric Product Specification), aiming at expressing the geometric product specifications. GPS, as any other language intended for tolerance specification, is strictly linked to the instruments available for geometric inspection. The GPS norms are mainly based on the inspection capabilities of CMM (Coordinates Measurement Machine), gauges, dial gauges, etc. The concepts of tolerance and dimension also depend on the scheme of representation adopted to specify the geometric entities. Currently, according to the GPS

standards, these concepts refer to a 2D representation obtained by projecting the objects orthogonally onto a plane. Nowadays, with the advent of high-resolution optical digitisers, new prospects are offered for real automatic geometric inspection [1] and, by extension, for tolerance specification [2]. The measurement process carried out by these digitisers allows of a 3D acquisition of the real object. The acquired points identify the surfaces of the object so it is possible to recognise from them the geometric properties from which to detect references for the evaluation of non-idealities. Thanks to the high-resolution digitisers new categories of nominal reference and tolerance can be conceived, which improves the signs of the traditional languages for geometric tolerance specification. Moreover, new procedures can be proposed in order to verify traditional tolerance categories.

For some years now, our research group has been developing a new methodology for automatic tolerance inspection starting from an acquired high-density 3D model. With a view to grouping together all the information recognisable in a scanned object, this paper presents a new data structure, called *Recognised Geometric Model (RGM)*. This geometric reference model is the result of an approach founded on the concepts of *non-ideal feature*

and *intrinsic nominal reference*. The object to be inspected is segmented into a set of *non-ideal features* and, for each of them, one or more *intrinsic nominal references* are identified. The recognition of the *intrinsic nominal references* from a scanned object is carried out based on some *rules* which, therefore, play a leading role in the definition of the domain of the representable entities within RGM. Based on these considerations, new categories for form non-idealities are introduced. However, this approach does not rule out the possibility of querying the RGM data structure by *explicit* geometric product specifications, in order to gather some quantitative information concerning special intrinsic geometric parameters and/or non-idealities. The new approach has been implemented in original software and tested for a real test case.

## 2 Literature review

Recent efforts have been made to develop methods which apply non-contact digitising techniques to geometric inspection. A great number of the methodologies proposed in literature require the knowledge of the CAD model of the workpiece under inspection. This model provides the nominal references in the form of analytical surfaces describing the geometric model ([1], [3], [4], [5], [6]). Since tolerance specifications usually refer to some features of the workpiece, the mapping between one surface (or feature) of the CAD model and the corresponding scanned point sub-cloud needs to be performed. Tolerance specifications can be either included in the CAD model as textual information, or interactively defined by the user. No standard language capable of specifying tolerances and suited to automatic verification has yet been defined, though. Prieto et al. in [3] propose and implement a methodology for the automated inspection of manufactured parts. Their methodology first registers the experimental point cloud with the corresponding CAD model of the workpiece by using the *Iterative Closest Point (ICP)* algorithm, and then segments the 3D point cloud by associating the points matching the same local geometric properties with the nearest CAD surface. This methodology is capable of verifying both dimensional and geometric tolerances. A similar approach is proposed by Li and Gu ([4], [5] and [6]). Gao in [1] develops an automated geometric inspection system within commercial RE software. The authors define a Nominal Inspection Frame (NIF) for a CAD model or a digitised reference model (Master Model) where every dimensional and geometric tolerance specification may be defined and interactively specified by the user. The GD&T items, which can be defined by the NIF, have to do with:

- flatness;
- datum (point, line, axis or plane);
- parallelism;
- perpendicularity;
- true position.

Once the NIF is created, the scanned data are then aligned with the nominal model; GD&T items on the measured parts are automatically computed and extracted from the NIF previously defined by means of the software functions and macro programs.

All these approaches evaluate the form non-idealities in the same way as the “profile tolerance of a surface” does [7] (the tolerance zone is limited by two CAD surfaces placed up and down the CAD reference surface at a distance  $t/2$  from it), without taking into account specific

geometric properties of the surface (for example, axially – symmetric surface, extruded surface, etc.). Some specific properties of the surface can actually play an important functional role in the object. For this purpose, ISO 1101 and ASME 14.5Y consider form tolerances, such as straightness, circularity, etc., which may be applied to derived or extracted features from the surface (axis, planar section of the surface, etc.). The approaches presented in literature which use a CAD model as an analytical reference show some limitations if compared with the traditional approach to tolerance inspection. In other words, the specification “language” based on these methods is poorer than that traditionally used by ISO or ANSI/ASME. Furthermore, these approaches do not take advantage of the specific way to inspect the real object or the numerical devices that can be used to evaluate the acquired point cloud. Based on these specific characteristics, new categories of form non-ideality can be introduced in accordance with the duality principle reported in GPS standards [8].

## 3 The RGM

RGM is an idealised geometric representation of a measured object deriving from the recognition of some ideal properties of the object. Figure 1 shows the flowchart for the process according to which the RGM is derived and queried. This representation is drawn from a high-density point cloud, which reproduces the real object being acquired. The recognition of these properties (form, orientation and location) is carried out based on some *rules*, which play a leading role in the RGM construction.

### 3.1 The RGM data structure

RGM is an idealised representation derived from an acquired real object. This geometric representation can be described by means of a hypergraph structure denoted by  $RGM(V, \varepsilon)$ , where  $V$  is the finite set of nodes  $v_i$  of the hypergraph. Each node is associated with a non-ideal feature of the measured object. Some labels are assigned to each node of the hypergraph, which describe the non-ideal feature attributes (such as the type of ideal feature). In  $RGM(V, \varepsilon)$ ,  $\varepsilon$  is a family of two sub-sets of  $V$  ( $\varepsilon = \{V_A, V_{GR}\}$ ) which are known as hyper-edges and which respectively represent the sets of non-ideal features for which adjacency relationships and mutual geometric properties are respectively recognised.

### 3.2 The non-ideal feature segmentation

At a first phase a complex segmentation process is directed toward the identification of *non-ideal features* and the associated category of recognisable ideal properties. We define a *non-ideal feature* as a set of adjacent points that are recognised to be smooth, of the same type (flat, umbilical, ruled and generic) and to pertain to a unique regular surface. The type of point is deduced by evaluating some differential geometric properties. In this paper, the segmentation process follows a surface hybrid approach based on fuzzy logic [9]. The set of *non-ideal features* represents the object with the exception of its non-regular parts (ridges and singularities). These non-regular parts do not follow a recognisable rule and are therefore excluded from further elaborations [10].

### 3.3 The ideal feature recognition

This recognition process is carried out based on some rules (here referred to as *recognition rules*) involving the evaluation of some local and global differential properties of the segmented model. The present work classifies the *ideal features* into two main categories: *analytical* and *non-analytical features*. *Analytical features* are those recognised to pertain to an analytical geometric surface (plane, sphere, cylinder, cone, etc.). For these *ideal features*, an analytical type of geometric surface is automatically recognised and associated with the *non-ideal feature*. Those *ideal features* which are not analytical are defined to be *non-analytical features*. *Non-analytical features* also include surfaces characterised by some specific and recognisable geometric properties. Based on the recurrence of specific differential geometric properties among the points pertaining to the *non-ideal*

*feature*, they can be classified as: generic extruded, generic cone, generic axially-symmetric. Henceforth we will be referring to them as *generic ruled (GR)* and *generic axially-symmetric (GA)*. The *non-analytical features* which cannot be included within any of the previous categories are *free form* features. In any case, for any non-analytical feature a parameterised equation can be associated with a CAD model by a registration process. In the *RGM* data structure the concept of *ideal feature* has to do with the qualitative property (or attribute) of a corresponding *non-ideal feature*: its geometric type (*analytical, GA, GR, free form*). The *ideal feature* itself does not identify quantitative elements, although some measurable (dimensionable) intrinsic characteristics (diameter, apex angle, etc.) and situation features (centre, axis, etc.) can be identified. Quantitative elements are identified during the querying phase of the *RGM*.

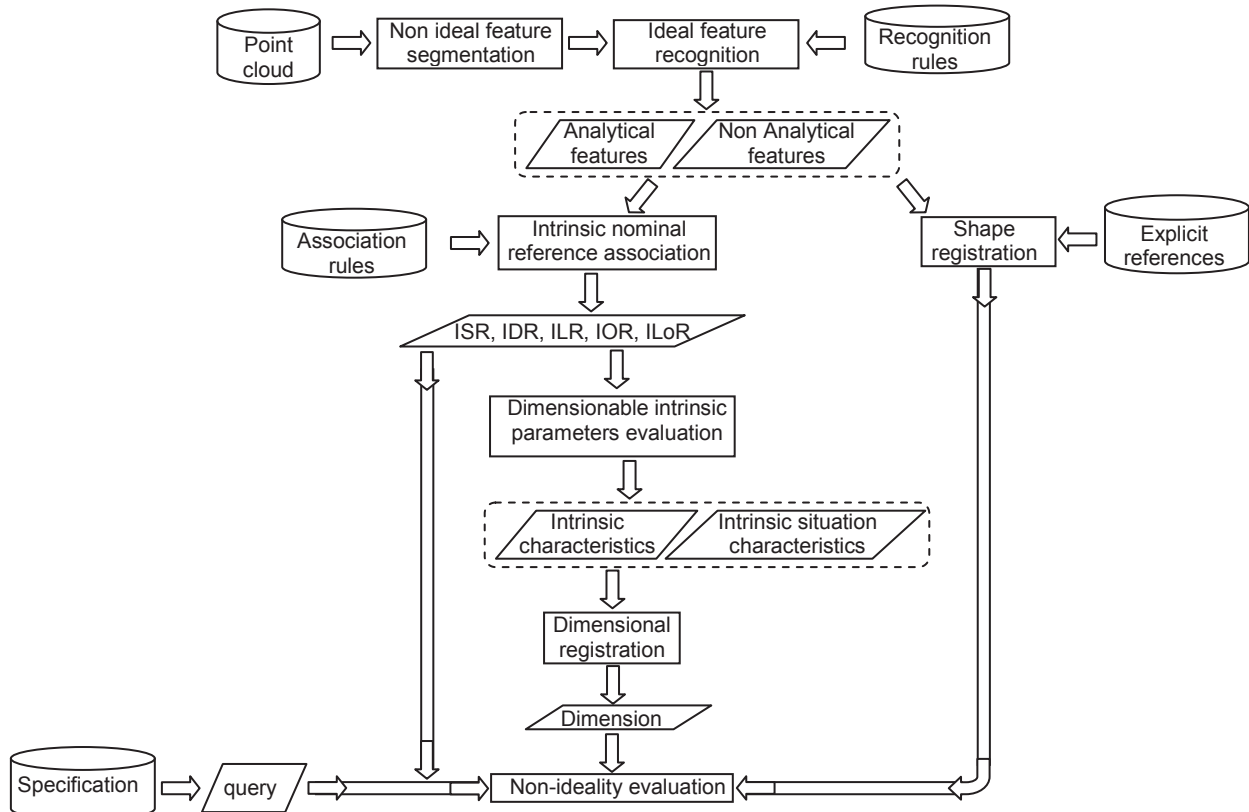


Fig 1: Flowchart for the process of derivation and query of the RGM

### 3.4 Intrinsic Nominal Reference Association

The *RGM* final aim is the evaluation of the *non-idealities* (form, orientation and location non-idealities) of the acquired object. This operation is performed by means of a query to the *RGM*. Non-ideality evaluation always requires the knowledge of a nominal (or ideal) reference. In a previous work [12] we classified the nominal references into two main categories: *explicit* and *intrinsic*. The *explicit* reference can be provided by a specification, also by using a CAD model through a registration process ([1], [3]). The *Intrinsic references* are nominal entities that are recognised in *non-ideal features* of the measured object. They can be classified as:

- *Intrinsic Shape Reference (ISR)*;
- *Intrinsic Derived References (IDR)*;
- *Intrinsic Local References (ILR)*;
- *Intrinsic Orientation References (IOR)*;
- *Intrinsic Location References (ILoR)*.

#### 3.4.1 The ISR association

An *ISR* is recognised in those points of the acquired workpiece which can be considered to be lying on an analytical surface (for example, plane, sphere, cylinder, cone, torus, etc.). The recognition of this type of reference requires the identification of an analytical surface, which approximates the points belonging to the tessellated surface. The *ISRs* are recognised by means of some rules (here referred to as *association rules*) which play a leading role in their definition. At present, GPS standards do not define these rules univocally. In literature several *association rules* are proposed. The most used of these aims at determining the nominal reference S (the ideal feature) best-fitting the related point cloud  $\mathbf{p}_i$  (non-ideal feature) by the  $L_\alpha$  - norm:

$$L_\alpha = \left[ \frac{1}{N} \sum_{i=1}^N |d(S, \mathbf{p}_i)|^\alpha \right]^{\frac{1}{\alpha}} \quad (1)$$

where  $0 < \alpha < \infty$ ,  $N$  is the total number of data points and  $d(S, p_i)$  is the shortest distance (or equivalently, the residual error) between  $p_i$  and  $S$ . Based on the  $\alpha$  value, several association rules are possible: the most used are summarised in table 1.

	Rule name	Rule expression
$\alpha=1$	L <sub>1</sub> -norm rule	$\sum_{i=1}^N  d(S, p_i) $
$\alpha=2$	L <sub>2</sub> -norm rule or least squares approach	$\sum_{i=1}^N d^2(S, p_i)$
$\alpha=\infty$	L <sub>∞</sub> -norm rule or min – max approach	$Max_{i=1, \dots, N} \{  d(S, p_i)  \}$

Tab. 1 The most used association rules for ISR association

Depending on the type of *ISR*, some dimensionable intrinsic geometric parameters, referred to as *Intrinsic Characteristics*, can be identified. Table 2 lists the *Intrinsic Characteristics* of the *ISRs* which can be recognised by the methods implemented in the present work.

Type of ISRs	Intrinsic Characteristics
Plane	none
Sphere	diameter
Cylinder	diameter
Cone	apex angle

Tab. 2 The Intrinsic Characteristics of the ISRs

For a *non-analytical feature* the *ISR* must be given explicitly.

### 3.4.2 The IDR association

An *IDR* is an analytically-known geometric entity deriving from the evaluation of some geometric properties of a feature. Generally speaking, these entities are not physical; for this reason, they are not directly measurable in the object, but can nevertheless be derived from the

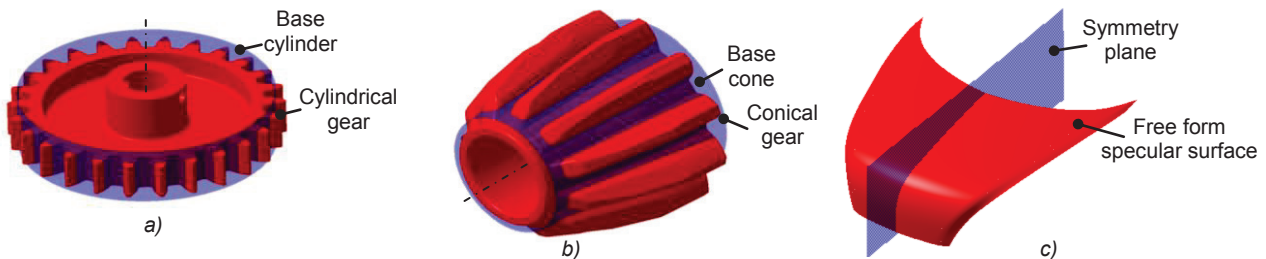


Fig 2: Examples of IDRs

### 3.4.3 The ILR association

*ILR* is an original type of nominal reference since it is not considered in the current tolerancing standards. It deals with the uniformity of some intrinsic differential geometric properties, such as: regularity, curvature recurrences, and so on. These references do not pertain to the global analytical properties of a surface, but rather to properties which locally characterise it. In what follows, two *ILRs* are introduced.

#### Profile Regularity

Roughly speaking, a regular profile is assimilable to a differentiable curve and any imperfection is associated with a deviation from it. This work takes the best local approximation of the data point with a regular curve as the nominal reference for profile regularity evaluation. The rules to identify the nominal reference include: the type of regular curve (polynomial curve, exponential curve, etc.) and the method to locally approximate the profile. In the

measured surface. Examples of *IDRs* are the axis and the circular sections of axially-symmetric surfaces. Whereas the circular sections are physical entities, the axis is not. In many cases these references are the situation features of the *non-ideal features*, since they serve as references for feature location and/or orientation. The situation features, for each type of *non-ideal feature* here considered, are reported in table 3. They are *ideal features* of the following types: point ( $\wp$ ), straight line ( $r$ ) or plane ( $\Pi$ ).

Type of non-ideal feature	Situation features
Plane	the plane $\Pi$ ;
Sphere	the centre $\wp$ ;
Cylinder	the axis $r$ ;
Cone	the axis $r$ ; the apex $\wp$ ;
Generic axially-symmetric	the axis $r$ ;
Generic extruded	the extrusion direction $r$ ;
Generic cone	the apex $\wp$ ;
Free form	-----

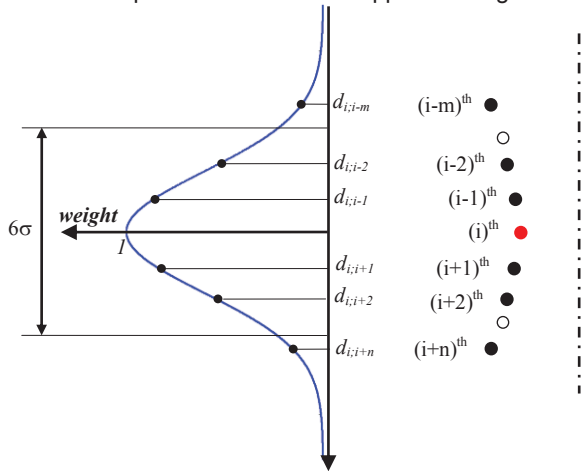
Tab. 3 The situation features for each type of non-ideal feature considered

Other *IDRs* could be defined based on functional or manufacturing properties of the acquired object. For the gears shown in figure 2, the base cylinder (figure 2 a) or the cone (figure 2 b) could represent the *IDRs* of the object and their axes could represent the situation feature of the gear. A further example could be the symmetry plane of a *free-form* mirrored surface (figure 2 c). The implementation of these *IDRs* in the *RGM* requires the introduction of specific recognition rules. The derived references are estimated by approximating the point cloud associated with a non-ideal feature by one or more association operations. Depending on the association rule which is used, different intrinsic references can be estimated.

approach herein proposed, the reference curve is evaluated, at each point of the profile, as the approximating curve of its neighbourhood. It is a quadric or a cubic polynomial, calculated by the weighted L<sub>2</sub>-norm rule. The weights of the approximation rule are assumed to be the values of a Gaussian function having the mean located at the analysed point and a properly selected value for the standard deviation  $\sigma$  (figure 3). This weighting approach aims at defining a regular profile by filtering the local irregularities. The width of this filter is conventionally assumed to be  $\lambda = 6\sigma$  of the Gaussian function. The  $\lambda$  value is assumed to be the maximum value between the expected maximum size of the profile imperfection and the mesh dimension. The profile regularity error, at each point analysed, is defined as the distance between the point and the related approximating curve.

**Surface Regularity**

Surface regularity is a generalisation (three – dimensional extension) of the profile regularity concept. A *non-ideal feature* consists of adjacent mesh vertices having some uniform differential geometric properties. Surface regularity measures locally the membership of each point to a regular surface which locally approximates its neighbourhood. Similarly to profile regularity, the rules to identify the nominal reference include: the type of regular surface and the method to locally approximate the surface. For each *non-ideal feature* recognised, the reference surface is evaluated at each vertex as the approximating surface by means of the *weighted* L<sub>2</sub>-norm rule. Surface regularity is not related to a specific shape of surface; it rather refers to the differentiability, which is a local property. Thus, in this work, the quadric paraboloid is used as a reference; it is the typical analytical regular surface used to evaluate the differential geometric properties. The weights of the approximation rule are assumed to be the values of a two – dimensional symmetric Gaussian function having the mean located at the analysed point and a properly selected value for the standard deviation (figure 4). In this case, the  $\lambda$  value is conventionally assumed to be equal to three times the maximum dimension of the mesh. The *surface regularity error*, at each point analysed, is defined as the distance between the point and the related approximating surface.



$d_{ij}$  is the signed distance between the  $i$ -th point and the  $j$ -th one  
**Fig. 3. The weighting approach to define profile regularity**

**Ruledness**

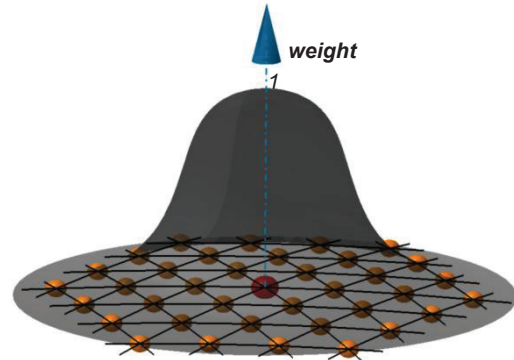
This *Intrinsic Local Reference* has to do with the confirmation of the ruled property of the acquired surface. It assumes an important role in generic ruled surfaces for which a reference of the type *ISR* cannot be identified. For this type of surfaces an *Intrinsic Derived Reference* is typically used. It is a straight line representing a surface generatrix and the related tolerance is the straightness tolerance [13]. This tolerance is adequate to the nominal concept of ruled surface, but is difficult to verify practically.

The rule to evaluate the related intrinsic reference, herein being proposed, is based on a typical growth algorithm. The intrinsic reference is an analytical ruled paraboloid whose form is expressed in the coordinate system  $(\xi, \psi, \zeta)$  as follows:

$$\zeta = c \cdot \psi^2 + d \cdot \xi + e \cdot \psi + f \quad (2)$$

Its parameters  $(c, d, e$  and  $f)$  are obtained by best fitting the point cloud around its vertex generatrix. For this purpose, a weighted approximation rule is used; the

weight factors are the values of a Gaussian function having the mean located in the vertex generatrix and a value for the standard deviation chosen according to the mesh dimensions ( $\sigma = \text{max mesh dimension}$ ). Since the generatrix direction is unknown, it must be sought by a growing algorithm from a seed point (figure 5).



**Fig. 4 The weighting approach to define surface regularity**

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Let  $\mathbf{p}_s$  be the seed point;
Let RS be the ruled surface;
Let  $NGH_s$  be the 1-ring neighbourhood of  $\mathbf{p}_s$ ;
Let  $L_{lim}$  be the specified limited length;
Let  $L_j$  be the generatrix length at the  $j$ -th step;

Let  $\Gamma_{ruled}$  be the ruled paraboloid locally approximating the mesh and expressed by equation (2);
Let  $\mathbf{x}_2$  be the estimated principal direction related to the null curvature;
Let  $\mathbf{p}_j$  be the nearest point  $\mathbf{p}_j$  from the generatrix along  $\mathbf{x}_2$  at the  $j$ -th step;
Let  $\Sigma_j$  be the set of points to approximate:

 $\Sigma_j = NGH_s \cup \{\mathbf{p}_i, i = 1, \dots, j\} \cup \{NGH_i, i = 1, \dots, j\}$ 

initialise  $j=0$ ;
initialise  $L_0=0$ ;

identification of  $\mathbf{p}_s \in RS$ ;
construction of  $NGH_s$ ;
Loop until ( $L_j < L_{lim}$ ) {
     $j=j+1$ ;
    calculation of  $\Gamma_{ruled}$ ;
    Re-evaluation of  $\mathbf{x}_2$  at  $\mathbf{p}_s$ ;
    Identification of  $\mathbf{p}_j$ ;
    Construction of  $\Sigma_j$ ;
    Evaluation of  $L_j$ }
Ruledness evaluation
    
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**Fig. 5. The ruledness evaluation algorithm**

The value of  $L_0$  must be included in the tolerance specification. Seed points are chosen so as to lie on the unique directrix curve of the surface. The *ruledness error* is evaluated as the distance between the last estimation of  $\Gamma_{ruled}$  and  $\mathbf{p}_s$  and the set of nearest points  $\mathbf{p}_j$ . More details are available in [12].

**3.4.4 The IOR association**

The *Intrinsic Orientation Reference (IOR)* refers to the mutual geometric properties of parallelism and perpendicularity and to some angular values which are recognised to be very close to values frequently occurring in mechanical workpieces, such as  $30^\circ$ ,  $45^\circ$  and  $60^\circ$ . These properties are recognisable between *non ideal features* whose situation features include a spatial direction, such as plane, cylinder, cone, GA and generic extruded. For the recognition of these properties it seems

more suitable to classify the features into planar features (henceforth referred to as *P – feature*) and features whose situation feature is an axis or an extrusion direction (henceforth referred to as *R – feature*). In the RGM a set of N parallel features of the same type constitutes a *system of homogeneous parallel entities* ( $S_{h//}$ ). Two types of systems can be identified: the system including *R – features*, which is denoted by  $S_{//R}$ , and that including *P – features*, which is denoted by  $S_{//P}$ . These systems can be automatically detected by analysing the *Mutual Parallelism Relationship Graph for R – features*  $G_{//R} = (V_{//R}, E_{//R})$  and the *Mutual Parallelism Relationship Graph for P – features*  $G_{//P} = (V_{//P}, E_{//P})$ , respectively.  $G_{//R}$

( $G_{//P}$ ) is a graph where each node represents an *R – feature* (*P – feature*) and each edge represents a mutual parallelism property recognised between *R – features* (*P – features*). In  $G_{//R}$  ( $G_{//P}$ ) there are as many *components* (i.e. maximal connected sub-graphs [15]) as spatial directions for which a parallelism property has been recognised. A component of  $G_{//R}$  ( $G_{//P}$ ), characterised by N nodes, is a *system of homogeneous parallel entities*  $S_{//R}$  ( $S_{//P}$ ) if the *degree* of each node is equal to N-1. For example, for the object shown in figure 6a, five  $S_{h//}$  systems are recognised (figure 6b). Each  $S_{h//}$  in  $G_{//P}$  or  $G_{//R}$  is associated with a *reference spatial direction*.

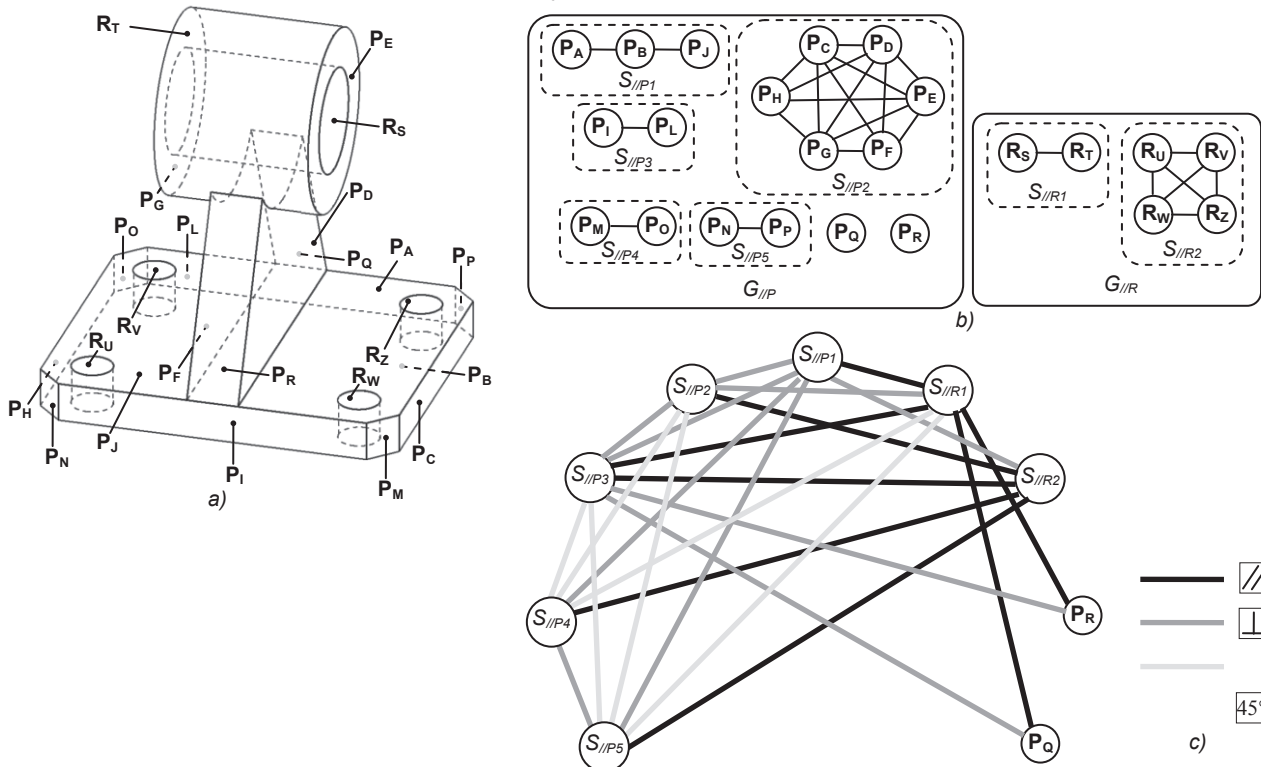


Fig. 6. An example of the recognition of orientation properties and related graphs

In order to recognise an intrinsic orientation property between non-ideal features, some rules must be defined first. In this work, the rules to recognise these properties are based on the evaluation of the dot product between the spatial directions of the *ideal features*. These rules vary depending on whether the features are of the same type (*P – features* or *R – features*) or of a different type (see table 4). Due to the non-ideality of real objects, the dot product never exactly matches the ideal value. Consequently, the recognition of the orientation property cannot be deduced from the mathematical verification of equality. In this paper, however, the values are considered to ‘match’ if the dot product value falls within a properly given tolerated range around the ideal value (reported in table 4 for the various properties). A further control consists in verifying the coherence of the parallelism relationships when these are recognised in pairs between N features ( $N \geq 3$ ) of the same type (*R – features* or *P – features*).

The transitive property, in practical cases, cannot be verified and ambiguities arise due to errors of *type I* and errors of *type II*. An error of *type I* occurs when the orientation property is true but nonetheless fails to be recognised. An error of *type II* originates when an orientation property is recognised between features, but is not true. This incoherence must be solved. For this

purpose, a first approach consists in applying the missing parallelism conditions so that the transitive property is verified. A second approach consists in splitting the incoherent set of parallel entities into two or more *systems of homogeneous parallel entities*. The first approach may give rise to errors of *type II*, whereas the second one may generate errors of *type I*. These criteria, which are to be adopted to solve the above-mentioned incoherence, are part of the parallelism recognition rule. In either case the criterion adopted produces uncertainties.

The Mutual Orientation Relationship Graph (MORG)

The concept of system of homogeneous parallel entities is functional to build the *Mutual Orientation Relationship Graph*  $MORG = (V_{OR}, E_{OR})$ . The nodes of MORG ( $V_{OR}$ ) are systems of homogeneous parallel entities  $S_{h//}$  or single features which do not belong to any  $S_{h//}$ . The arcs ( $E_{OR}$ ) identify mutual orientation properties between connected nodes. A label is assigned to each edge which specifies the type of recognised orientation (parallelism, perpendicularity, 30°, 45° and 60°). The MORG in Figure 6c refers to the object shown in figure 6a.

Thanks to the concept of system of homogeneous parallel entities, if an orientation property is recognised between one feature of a system  $S_{h//1}$  and one feature of a system  $S_{h//2}$ , then it is possible to deduce the satisfaction of this

property between each feature of  $S_{h/1}$  and each feature of  $S_{h/2}$ .

dot product value		Intrinsic Orientation Property
Features of the same type	Features of a different type	
0	1	perpendicularity
$\pm 0.5$	$\pm \frac{\sqrt{3}}{2}$	angular orientation of 60°, -60°, 120° and 240°
$\pm \frac{\sqrt{2}}{2}$	$\pm \frac{\sqrt{2}}{2}$	angular orientation of 45°, -45°, 135° and 225°
$\pm \frac{\sqrt{3}}{2}$	$\pm 0.5$	angular orientation of 30°, -30°, 150° and 210°
$\pm 1$	0	parallelism

Tab. 4 The rules for intrinsic orientation property recognition

### 3.4.5 The Intrinsic Location Reference association

The *Intrinsic Location Reference (ILoR)* includes special types of location relationships. They are: concentricity, coaxiality and coincidence. The recognition of these intrinsic mutual location relationships requires that a *localised* situation feature should be identified for each *non-ideal feature*. A situation feature is said to be localised if it is possible to unequivocally identify its location within an arbitrarily given reference frame. For example, the axis of a GA localises the surface to a plane which is orthogonal to the axis. On the contrary, the extrusion direction, that is to say, the situation feature recognised for a generic extruded surface, is not a localised situation feature.

Table 5 shows the intrinsic location properties which have been recognised. In order to detect them some *recognition rules* must be defined first.

Coaxiality is the property of two ideal axially-symmetric features having the same axis. The rule which is here being followed involves first the parallelism recognition

and then the evaluation of the related distance between the axes of the features. Coaxiality is recognised if the distance value (estimated by approximating the axially-symmetric features by the  $L_2$ -rule) falls within a properly given tolerated range around 0.

In the GPS standards, the concept of concentricity is defined between features projected onto a plane [7]. In this work this concept is extended to include 3D features. Concentricity is then the property of two spherical ideal features having the same centre. As far as the rule for the recognition of this property is concerned, it should be based on the comparison of the distance between the centres of the two spheres (analytically identified by the  $L_2$ -rule) with a properly given tolerated value.

Coincidence is the property of two features which are one in the continuation of the other. Some recognition rules are also defined in order to detect this property between some analytical features (plane, cone, cylinder, sphere). These rules vary depending on the non-ideal feature type. For analytical axially – symmetric features (cone and cylinder) the rule involves first the coaxiality-property recognition and then the evaluation of the difference between the corresponding intrinsic characteristics (estimated by approximating the features by the  $L_2$ -rule). If said difference value falls within a properly given tolerated range around 0, the features can be recognised to be coincident with each other. An analogous recognition rule can be defined for spherical features. In this case, the pre-emptive recognition of the concentricity property, rather than the coaxiality property, is required. For planar features the recognition rule involves first the parallelism detection and then the evaluation of the corresponding *intrinsic location characteristic* (representing the distance between the planar features). If this value (estimated by approximating the features by the  $L_2$ -rule) falls within a properly given tolerated range around 0, the coincidence property is recognised (table 6).

Feature type	Plane	Sphere	Cylinder	Cone	Generic Axially – symmetric (GA)
Plane	Coincidence				
Sphere	-	Concentricity Coincidence	-	-	
Cylinder	-	-	Coaxiality Coincidence	Coaxiality	Coaxiality
Cone	-	-	Coaxiality	Coaxiality Coincidence	Coaxiality
Generic Axially – symmetric (GA)			Coaxiality	Coaxiality	Coaxiality

Tab. 5 The intrinsic location properties recognised for the several feature types

Type of analytical features	Required mutual geometric relationship	Further condition to be verified
Plane	Parallelism	Distance between planar features
Sphere	Concentricity	Difference between diameters
Cylinder	Coaxiality	Difference between diameters
Cone	Coaxiality	Difference between apex angles

Tab. 6 The coincidence property recognition

When intrinsic location properties of a specific type (concentricity, coaxiality or coincidence) are recognised in pairs between N features ( $N \geq 3$ ), the transitive property must be verified. Any set of N features for which

$\sum_{i=1}^N (N - i)$  location properties are recognised in pairs is said to automatically satisfy the coherence imposed by the transitive property. This set of features is referred to as *system of coherent localised entities* ( $S_{Lo}$ ).

#### The Mutual Location Relationship Graph (MLRG)

In order to represent intrinsic location properties and identify the different types of *system of coherent localised*

*entities*, a specific graph is defined. The *mutual location relationship graph MLRG* = ( $V_{LR}$ ,  $E_{LR}$ ) is a graph where each node identifies mutual location properties between connected nodes. Each edge is assigned a label which reports the type of location recognised (coaxiality, concentricity and coincidence).

The *systems of coherent localised entities* can be automatically detected starting from this graph. In *MLRG* there are as many *components* as non-ideal features for which an intrinsic location property has been recognised. A component, characterised by N nodes, represents a *system of coherent localised entities*  $S_{Lo}$  (for a specific type of location property) if the *degree* of each node is

equal to  $N-1$ . If the degree of any nodes of the component is less than  $N-1$ , the coherence verification, imposed by the need to satisfy the transitive property, cannot be possibly carried out. The subsequent incoherence could be solved either by imposing the missing location conditions or by splitting the component into two or more *systems of coherent localised entities*  $S_{Lo}$ . The first approach could give rise to errors of *type II*, whereas the second one could generate errors of *type I*.

A *reference situation feature* is associated with each  $S_{Lo}$ . This turns out to be useful during the phase of RGM query whenever the user asks for the evaluation of the intrinsic situation characteristics between two non-ideal features. If one or both non-ideal features belong to a system of *coherent localised entities*  $S_{Lo}$ , it is possible to consider the reference situation feature (associated with  $S_{Lo}$ ) rather than the situation feature of the single feature. This allows us to obtain a value for the situation characteristic which is coherent with satisfying the transitive property.

### 3.4.6 Dimensionable intrinsic parameters' evaluation

RGM includes several dimensionable geometric parameters. They are both the *intrinsic characteristics* and the *intrinsic situation characteristics*. According to the definition given by the GPS standards, *situation parameters* describe "the relative situation (location or orientation) between two situation features" [16]. They can be further divided into *location parameters* and *orientation parameters*. The former are expressed by length (distance) values; the latter are expressed by angular values. In RGM an *intrinsic location characteristic* is automatically associated whenever a parallelism property is recognised between two localised situation features. The dimensionability of the parallelism is therefore an intrinsic reference. This dimensionable characteristic does not identify an intrinsic reference value for the dimension, but a reference dimension can nevertheless be specified for it. This operation is here referred to as *dimensional registration* (figure 1).

The *intrinsic location characteristic* represents the distance between two parallel ideal entities approximating the two situation features recognised to be parallel to each other. In an ideal model, the coherence between the distances of three or more situation features must be verified in order to satisfy the non-contradiction principle so that:  $d_{A-B} + d_{B-C} = d_{A-C}$ . On the contrary, in an acquired object in which the features are recognised to be parallel despite the fact that they are not really parallel to each other, every dimension can be evaluated independently from the others. For this reason,  $d_{A-B} + d_{B-C} \neq d_{A-C}$  and the representation of the three distances are not really a redundancy.

In the RGM query, several values for the *Intrinsic Location* parameter can be obtained depending on the association rule used for approximating the two *non-ideal features* (for example, the least squares fitting  $L_2$ , the upper or inner envelope fitting, etc.).

#### Intrinsic Location Parameters Graph (ILPG)

In order to represent the *intrinsic location characteristics* in the RGM, a specific graph is built which is referred to as *Intrinsic Location Parameters Graph (ILPG)*. Thanks to the concept of *system of coherent localised entities*, this graph can be efficiently defined. The nodes of ILPG are *systems of coherent localised entities*  $S_{Lo}$  or single features which do not belong to any  $S_{Lo}$ . In either case

the features can be planar or axially-symmetric since the intrinsic location characteristic can only be defined for these types. The arcs of ILPG represent the intrinsic location parameters identified between the connected nodes. According to the concept of *system of coherent localised entities*, if an intrinsic location parameter is recognised to exist between one feature of a system  $S_{Lo1}$  and one feature of a system  $S_{Lo2}$ , an unequivocally identified dimensional parameter can be associated between each feature of  $S_{Lo1}$  and each feature of  $S_{Lo}$ . The identification of this single intrinsic location parameter is coherent with satisfying the transitive property.

### 3.5 Non-Ideality evaluation

The final phase of the methodology herein being proposed is error evaluation. For each type of *non-ideal feature* (see table 4), specific categories of form non-idealities can be identified. These are the categories that can be automatically evaluated as deviations from the recognisable intrinsic references. Table 7 does not show the curve or the surface profile tolerances, which rather require an explicit nominal reference should be specified.

All the form non-idealities can be expressed both as the maximum (*Max*) and as the standard deviation ( $\sigma$ ) of the point cloud distances from the corresponding intrinsic reference. A probabilistic evaluation of non-idealities can be more significant than the maximum deviation measure. The point cloud acquisition process is affected by singular errors which can be ascribed to the typical measuring errors affecting the optical scanner devices. The  $\sigma$  value performs a probabilistic estimation of the location of the acquired point with respect to the intrinsic nominal reference. This way to specify non-idealities is possible thanks to the large set of points acquired for each *non-ideal feature*. More details are reported in [12].

## 4

The methodology described in the previous sections has been implemented in original software, coded in C++, by using a library dedicated to the processing of tessellated geometric models, which has been developed at the University of L'Aquila. In order to verify the reliability of the proposed methodology, a specific test case has been analysed which refers to a real object whose acquisition has been carried out by means of an optical scanner ([www.scansystems.it](http://www.scansystems.it)). Figure 7a shows the results for the features identification and their respective labels. The areas coloured black are recognised to be non-regular and are therefore automatically excluded from the non-ideality evaluation. The situation features are all evaluated by using the  $L_2$ -rule [12]. Figure 7b shows the results obtained for the  $S_{h/l}$  recognition for the test case considered by using the software here implemented. In particular, the features belonging to each  $S_{//D}$  and  $S_{//R}$  are respectively listed with the corresponding *reference spatial direction*. Figures 7c and 7d display the graphs resulting from the recognition of orientation (figure 6c) and location (figure 7d) properties for the test case being considered. Finally figure 6e displays the report of the form error analysis. The results reported confirm the important role that the rules used to estimate the intrinsic references play in error evaluation.











## 5 Con□□□□ion

This paper proposes the idealised geometric representation called RGM (*Recognised Geometric*



Model) of an acquired high-density workpiece. RGM construction involves a complex segmentation process directed toward the identification of the *non-ideal features* and the associated category of recognisable ideal properties. The object to be inspected is segmented into a set of *non-ideal features* and, for each of them, one or more *intrinsic nominal references* are identified. The recognition of these references is carried out based on some *rules*, which play a leading role in the definition of the domain of the representable entities within RGM. Based on these considerations, new and old categories of form non-idealities are here defined and some procedures are proposed for a more robust process of verification of

traditional tolerance categories (such as the straightness of a cylinder generatrix). When using the RGM, tolerances can be specified according to the set of available and recognisable intrinsic nominal references. This allows for the automatic geometric inspection of the workpiece. However, the approach being proposed does not rule out the possibility of querying the RGM data structure by explicit geometric product specifications, in order to gather some quantitative information concerning special intrinsic geometric parameters and/or non-idealities. Future work should address how to specify the tolerated errors in accordance with the RGM data structure and how to measure the non-idealities of the object.

		type of feature								
		plane	sphere	cylinder	cone	generic axially-symmetric	generic extruded	generic cone	free form	
Form error	<b>Straightness</b>		--	--	Extracted median line	Extracted median line	Extracted median line	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
	<b>Flatness</b>		X	--	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
	<b>Roundness</b>		--	--	Any extracted cross-sectional circumferential line	Any extracted cross-sectional circumferential line	Any extracted cross-sectional circumferential line	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
	<b>Total Roundness</b>		--	--	X	X	X	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
	<b>Cylindricity</b>		--	--	X	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
	<b>Conicity</b>		--	--	<input type="checkbox"/>	X	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
	<b>Sphericity</b>		X	--	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Regularity error	<b>Profile</b>		X	X	X	X	X	X	<input type="checkbox"/>	<input type="checkbox"/>
	<b>Surface</b>		<input type="checkbox"/>	<input type="checkbox"/>	X	X	X	X	X	X
<b>Ruledness non-ideality</b>			--	--	X	X	--	X	X	--

Tab. 7 Types of non-ideal feature and related errors

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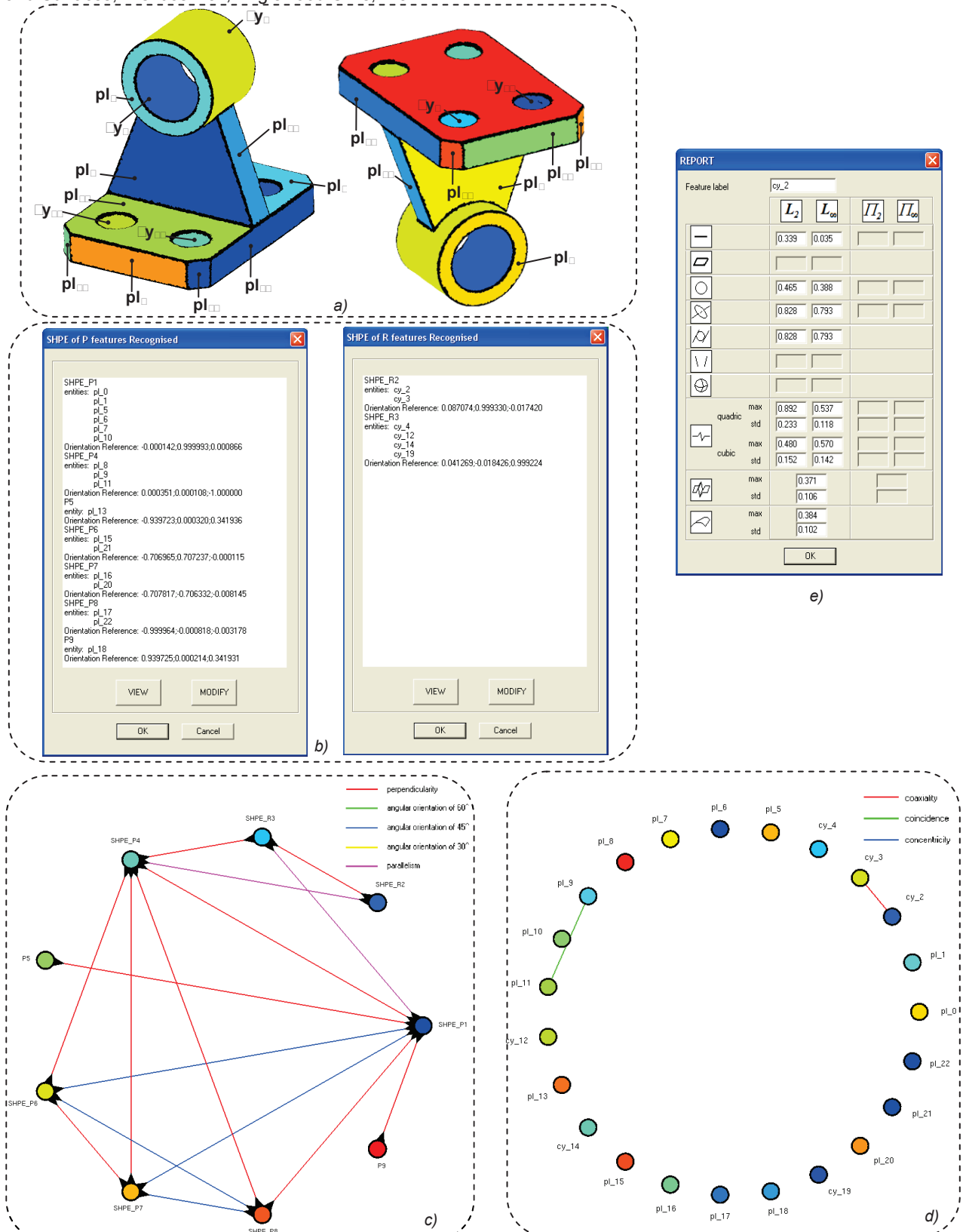


Fig. 7. The results of RGM construction for the test case under examination: a) features identification and their respective labels, b)  $S_{h/r}$  recognition, c) MORG resulting, d) MLRG resulting, e) report of the form error analysis.