

Es. 1

$$a) \int_0^{\pi/2} x \cos 2x \, dx = \frac{1}{2} x \sin 2x \Big|_0^{\pi/2} - \frac{1}{2} \int_0^{\pi/2} \sin 2x \, dx =$$

$$= \frac{1}{2} \cdot \frac{1}{2} \cos 2x \Big|_0^{\pi/2} = \frac{1}{4} (-1 - 1) = -\frac{1}{2}$$

$$b) I = \int_0^1 e^{-x} |2x-1| \, dx = \int_0^{1/2} e^{-x} (1-2x) \, dx + \int_{1/2}^1 e^{-x} (2x-1) \, dx$$

$$\text{ho usato } |2x-1| = \begin{cases} 1-2x & \text{se } 2x-1 < 0 \\ 2x-1 & \text{se } 2x-1 > 0 \end{cases}$$

$$\int e^{-x} (2x-1) \, dx = 2 \int e^{-x} x \, dx - \int e^{-x} \, dx =$$

$$= 2[-x e^{-x} + \int e^{-x} \, dx] - \int e^{-x} \, dx = -2x e^{-x} + \int e^{-x} \, dx$$

$$= -2x e^{-x} - e^{-x} = -(2x+1)e^{-x}$$

$$\Rightarrow I = (2x+1)e^{-x} \Big|_0^{1/2} - (2x+1)e^{-x} \Big|_{1/2}^1 =$$

$$= 2e^{-1/2} - 1 - (3e^{-1} - 2e^{-1/2}) = \frac{4}{\sqrt{e}} - 1 - \frac{3}{e}$$

$$c) \int \frac{2x}{x^2+2x-3} \, dx = \int \frac{2x}{(x-1)(x+3)} \, dx = \int \left[\frac{1}{2} \frac{1}{x-1} + \frac{3}{2} \frac{1}{x+3} \right] \, dx$$

$$= \frac{1}{2} \lg|x-1| + \frac{3}{2} \lg|x+3| + c$$