SCHEMA SOLUZIONE ANALISI 1 PARTE 2 25.1.2016

1) $$\sum_{n=1}^{\infty} \ln\left(\frac{n^4}{n^4+5n^2-1}\right) \quad (*)$$

$$\lim_{n\to\infty} \ln\left(\frac{n^4}{n^4+5n^2-1}\right) = \ln 1 = 0 \qquad \text{CN ok}$$

$$\ln\left(\frac{n^4}{n^4+5n^2-1} - 1 + 1\right) = \ln\left(1 + \frac{n^4-n^4-5n^2+1}{n^4+5n^2-1}\right) =$$

$$= \ln\left(1 + \frac{1-5n^2}{n^4+5n^2}\right) \underset{n\to\infty}{\sim} \frac{1-5n^2}{n^4+5n^2} \sim -\frac{5}{n^2}$$

$\sum_{n=1}^{\infty} \frac{5}{n^2}$ converge $\Rightarrow$ la serie $(*)$ converge

$$\sum_{n=0}^{\infty} \frac{n+5^n}{n^3+n!}$$

$$\frac{n+5^n}{n^3+n!} \underset{n\to\infty}{\sim} \frac{5^n}{n!} \xrightarrow[n\to\infty]{} 0 \qquad \text{CN ok}$$

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)+5^{n+1}}{(n+1)^3+(n+1)!} \cdot \frac{n+n!}{n+5^n} \underset{n\to\infty}{\sim} \frac{5^{n+1}\, n!}{(n+1)!\, 5^n} = \frac{5}{n+1} \xrightarrow[n\to\infty]{} 0 < 1$$

$\Rightarrow$ la serie converge