2)
$$\int_1^e x^2(\ln x)^3\,dx =$$

$$\frac{1}{3}x^3(\ln x)^3\Big|_1^e - \frac{1}{\cancel{3}}\int_1^e x^{\cancel{3}\,2}\,\cancel{3}(\ln x)^2\frac{1}{\cancel{x}}\,dx =$$

$$= \frac{e^3}{3}(\ln e)^3 - \frac{1}{3}x^3(\ln x)^2\Big|_1^e + \int_1^e \frac{1}{3}x^{\cancel{3}\,2}\,2\ln x\,\frac{1}{\cancel{x}}\,dx =$$

$$= \frac{e^3}{3} - \frac{e^3}{3} + \frac{2}{3}\cdot\frac{1}{3}x^3\ln x\Big|_1^e - \int_1^e \frac{2}{9}x^{\cancel{3}\,2}\frac{1}{\cancel{x}}\,dx =$$

$$= \frac{2}{9}e^3 - \frac{2}{27}(e^3-1) = \frac{4}{27}e^3 + \frac{2}{27}$$

$$\int \frac{x^3+2}{x^3-x}\,dx = \int \frac{x^3-x+x+2}{x^3-x}\,dx = \int\left(1+\frac{x+2}{x^3-x}\right)dx$$

$$\frac{x+2}{x(x^2-1)} = \frac{Ax+B}{x^2-1} + \frac{C}{x} = \frac{Ax^2+Bx+Cx^2-C}{(x^2-1)x} \quad (*)$$

$A+C=0 \quad A=2$

$B=1$

$C=-2$

$$\Rightarrow \frac{x+2}{x(x^2-1)} = \frac{2x+1}{x^2-1} - \frac{2}{x} = \frac{2x}{x^2-1} - \frac{2}{x} + \frac{1}{x^2-1}$$

$$\frac{1}{x^2-1} = \frac{1}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right) \Rightarrow \frac{x+1}{x(x^2-1)} = \frac{2x}{x^2-1} - \frac{2}{x} + \frac{1}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)$$

$$= \int\left(1 + \frac{2x}{x^2-1} - \frac{2}{x} + \frac{1}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)\right)dx = x + \lg|x^2-1| - 2\lg|x| + \frac{1}{2}(\lg|x-1| - \lg|x+1|) + c$$

$$= x + \lg\frac{|x^2-1|}{x^2} + \frac{1}{2}\lg\frac{|x-1|}{|x+1|} + c$$

(*) si poteva anche scrivere $\frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$