4) $\int_a^b f(x)g''(x)\,dx \qquad f(a)=f(b)=g(a)=g(b)=0$

$$= f(x)g'(x)\Big|_a^b - \int_a^b f'(x)g'(x)\,dx = -f'(x)g(x)\Big|_a^b + \int_a^b f''(x)g(x)\,dx$$

($f(x)g'(x)\Big|_a^b = 0$, $f'(x)g(x)\Big|_a^b = 0$)

$$= \int_a^b f''(x)g(x)\,dx \qquad \text{c.v.d}$$

5) $y' + \frac{2(t+1)}{t^2+2t+2}y = \frac{1}{(t+1)(t^2+2t+2)}$ $\qquad a(t) = \frac{2(t+1)}{t^2+2t+2}$ $\quad t\in\mathbb{R}$

$t^2+2t+2 \neq 0 \quad \forall t\in\mathbb{R}$

$t^2+2t+2 > 0 \quad \forall t\in\mathbb{R}$

$f(t) = \frac{1}{(t+1)(t^2+2t+2)}$ $\quad t\neq -1$

Soluzione generale $\qquad D = (-\infty,-1)\cup(-1,+\infty)$

$$A(t) = \int \frac{2(t+1)}{t^2+2t+2}\,dt = \lg(t^2+2t+2)$$

$$y(t) = e^{-A(t)}\left(c + \int f(t)e^{A(t)}\,dt\right)$$

$$e^{-A(t)} = e^{-\lg(t^2+2t+2)} = \frac{1}{t^2+2t+2}, \qquad e^{A(t)} = t^2+2t+2$$

$$\int f(t)e^{A(t)}\,dt = \int \frac{1}{(t+1)(t^2+2t+2)}(t^2+2t+2)\,dt = \int \frac{dt}{t+1} = \lg|t+1|$$

$$y(t) = \frac{1}{t^2+2t+2}\left(c + \lg|t+1|\right) \quad \text{soluzione generale}$$

$$y(-2) = 0 \qquad \frac{1}{4-4+2}(c + \lg|-2+1|) = 0 \qquad c = 0$$

$$y(t) = \frac{1}{t^2+2t+2}\lg|t+1| \quad \text{sol pb di Cauchy definita su } (-\infty,-1)$$