Es. 1

a) $\sum_{n=1}^{\infty}\left(e^{\frac{n+1}{3-n^2}}-e^{\frac{1}{n}}\right)$

C.N. $\lim_{n\to\infty} e^{\frac{n+1}{3-n^2}}-e^{1/n} = 1-1=0$

$$e^{\frac{n+1}{3-n^2}}-e^{1/n} = e^{\frac{1}{n}}\left(e^{\frac{n+1}{3-n^2}-\frac{1}{n}}-1\right) \underset{n\to\infty}{\sim} -\frac{2}{n}$$ uso $e^x\sim 1+x$, $x\to 0$

$$\frac{n+1}{3-n^2}-\frac{1}{n} = \frac{n(n+1)-3+n^2}{n(3-n^2)} \underset{n\to\infty}{\sim} -\frac{2}{n}$$

$\sum_n -\frac{2}{n}$ diverge $\Rightarrow$ la serie data diverge

b) $\sum_{n=1}^{\infty} (n+3)\sin\left(\frac{1}{n^4}\right)$

Studio $\sum_{n=1}^{\infty}\left|(n+3)\sin\left(\frac{1}{n^4}\right)\right|$

Dato che $\sin\left(\frac{1}{n^4}\right)\underset{n\to\infty}{\sim}\frac{1}{n^4}$ $\Rightarrow$ $\left|(n+3)\sin\left(\frac{1}{n^4}\right)\right| \underset{n\to\infty}{\sim} \frac{n+3}{n^4} \sim \frac{1}{n^3}$

$\sum_n \frac{1}{n^3}$ converge $\Rightarrow$ la serie data converge assolutamente $\Rightarrow$ converge.