Es. 6

$y' + 2xy = 2x \sin x^2$ $\qquad y' + a(x)y = f(x)$

$A(x) = \int 2x\,dx = x^2 \qquad y(x) = e^{-A(x)}\left(c + \int e^{A(x)} f(x)\,dx\right)$

$A(x) = \int a(x)\,dx$

$$y(x) = e^{-x^2}\left(c + \int e^{x^2} 2x \sin(x^2)\,dx\right)$$

$$\int e^{x^2} 2x \sin(x^2)\,dx = \int e^t \sin t\,dt$$

$t = x^2 \quad dt = 2x\,dx$

$$\int e^t \sin t\,dt = e^t(-\cos t) + \int e^t \cos t\,dt =$$

$$= -\cos t\, e^t + e^t \sin t - \int e^t \sin t\,dt$$

Se pongo $I = \int e^t \sin t\,dt$ ho ottenuto l'equazione

$$I = -e^t \cos t + e^t \sin t - I$$

$$\Rightarrow 2I = e^t(\sin t - \cos t) \iff I = \frac{e^t(\sin t - \cos t)}{2}$$

$y(x) = e^{-x^2}\left(c + e^{x^2}\dfrac{\sin(x^2) - \cos(x^2)}{2}\right)$ Integrale generale

$y(0) = 1 \qquad 1 = c - \frac{1}{2} \quad \Rightarrow c = 3/2$

Soluzione Pb. di Cauchy

$$y(x) = e^{-x^2}\left(\frac{3}{2} + e^{x^2}\frac{\sin(x^2) - \cos(x^2)}{2}\right)$$

$$y(x) = \frac{3}{2} e^{-x^2} + \frac{\sin(x^2) - \cos(x^2)}{2}$$