SCHEMA SOLUZIONE 1 PARTE 8.6.2015

Es. 1 $\quad y=\sqrt{3}\,x^{3/2}$

$$\vec r:\begin{cases}x=t\\ y=\sqrt{3}\,t^{3/2}\end{cases}\qquad t\in[0,3]$$

$$\vec r\,':\begin{cases}x'=1\\ y'=\sqrt{3}\,\frac{3}{2}\,t^{1/2}\end{cases}$$

$$ds=|\vec r\,'(t)|\,dt \qquad |\vec r\,'(t)|=\sqrt{1+\frac{27}{4}t}$$

$$L=\int_\gamma ds=\int_0^3\sqrt{1+\frac{27}{4}t}\,dt=\frac{2}{3}\left(1+\frac{27}{4}t\right)^{3/2}\frac{4}{27}\Bigg|_0^3$$

$$=\frac{8}{81}\left[\left(1+\frac{81}{4}\right)^{3/2}-1\right]$$

Es. 2 $\quad f(x,y)=\begin{cases}\dfrac{3xy}{x^2+y^2}\,\sin(x^2+y^2)^\alpha & (x,y)\neq(0,0)\\[2mm] 0 & (x,y)=(0,0)\end{cases}$

$\alpha>0$

$$|f(x,y)|\le\frac{|3xy|}{x^2+y^2}\,|\sin(x^2+y^2)|^\alpha$$

passo a coordinate polari

$$|f(\rho\cos\theta,\rho\sin\theta)|=\frac{3\rho^2|\cos\theta\sin\theta|}{\rho^2}\,|(\sin\rho^2)^\alpha|\le 3|\sin\rho^2|^\alpha \xrightarrow[\rho\to0]{} 0$$

$\Rightarrow$ se $\alpha>0$ la funzione è continua in $(0,0)$

$\alpha=0 \qquad f(x,y)=\dfrac{3xy}{x^2+y^2} \qquad f(x,x)=\dfrac{3x^2}{2x^2}=3/2$

$$f(x,-x)=-\frac{3x^2}{2x^2}=-3/2$$

$\Rightarrow$ il limite non esiste $\Rightarrow f$ non è continua in $(0,0)$