ANALISI MATEMATICA 2

ESAME 13.6 2016 1ª PARTE

SCHEMA SOLUZIONE

Es. 1 $\quad y'' - y' + 2y = 2e^{x}\cos 3x$

$z'' - z' + 2z = 0 \qquad \lambda^2 - \lambda + 2 = 0 \qquad \lambda = \frac{1 \pm \sqrt{1-8}}{2} = \frac{1 \pm i\sqrt{7}}{2}$

$z(x) = e^{\frac{1}{2}x}\left[c_1 \cos\left(\frac{\sqrt{7}}{2}x\right) + c_2 \operatorname{sen}\left(\frac{\sqrt{7}}{2}x\right)\right]$ soluzioni omogenea

Trovo soluzione particolare dell' eq. $y'' - y' + 2y = 2e^{(1+3i)x}$

$$y(x) = \frac{2e^{(1+3i)x}}{(1+3i)^2 - (1+3i) + 2} = \frac{2e^{(1+3i)x}}{1-9+6i-1-3i+2} = \frac{2e^{(1+3i)x}}{-7+3i} = \frac{2}{49+9}(-7-3i)e^{(1+3i)x}$$

$$y(x) = -\frac{2}{58}(7+3i)e^{(\,)} = -\frac{1}{29}(7+3i)e^{(1+3i)x} = -\frac{1}{29}(7+3i)e^{x}\left[\cos 3x + i\operatorname{sen} 3x\right]$$

$$\mathrm{Re}(y(x)) = -\frac{e^{x}}{29}(7\cos 3x - 3\operatorname{sen} 3x)$$

$$y(x) = e^{\frac{1}{2}x}\left[c_1\cos\left(\frac{\sqrt{7}}{2}x\right) + c_2\operatorname{sen}\left(\frac{\sqrt{7}}{2}x\right)\right] - \frac{e^{x}}{29}\left(7\cos(3x) - 3\operatorname{sen}(3x)\right)$$

INTEGRALE GENERALE

$y(0) = c_1 - \frac{7}{29} = 0 \qquad c_1 = 7/29$

$y'(0) = \frac{1}{2}c_1 - \frac{7}{29} + \frac{\sqrt{7}}{2}c_2 + \frac{9}{29} = 1 \Rightarrow -\frac{7}{29}\,\frac{1}{2} + \frac{\sqrt{7}}{2}c_2 + \frac{9}{29} = 1$

$\frac{\sqrt{7}}{2}c_2 = 1 - \frac{11}{2\cdot 29} \qquad c_2 = \frac{58-11}{58}\cdot\frac{2}{\sqrt{7}} = \frac{47}{29\sqrt{7}}$