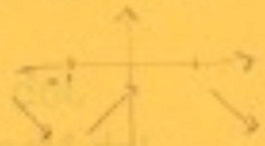
Es. 2 $\quad \vec{r}(t) = \left(\frac{2t}{1+t^2}, \frac{2t^3}{1+t^2} \right) \quad t \in \mathbb{R}$

$\lim_{t \to -\infty} \vec{r}(t) = (0, -\infty) \neq \lim_{t \to +\infty} \vec{r}(t) = (0, +\infty) \Rightarrow$ non chiusa

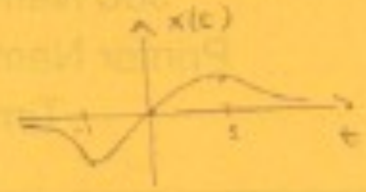
$$\begin{cases} x'(t) = \frac{2}{1+t^2} - \frac{4t^2}{(1+t^2)^2} = \frac{2(1+t^2) - 4t^2}{(1+t^2)^2} = \frac{2(1-t^2)}{(1+t^2)^2} \\ y'(t) = \frac{6t^2}{1+t^2} - \frac{4t^4}{(1+t^2)^2} = \frac{6t^2(1+t^2) - 4t^4}{(1+t^2)^2} = \frac{6t^2 + 2t^4}{(1+t^2)^2} > 0 \end{cases}$$

$x'(t) = 0 \iff t = \pm 1 \quad y(1) \neq 0 \quad y(-1) \neq 0$



$\Rightarrow \vec{r}(t) \in C^1(\mathbb{R})$ inoltre $\vec{r}'(t) \neq 0 \ \forall t \in \mathbb{R}$

$\Rightarrow$ la curva è regolare $\forall t$



$-1 \leq x(t) \leq 1 \qquad x(-1) = -1 \quad x(1) = 1$

$-\infty \leq y(t) \leq +\infty \qquad \Rightarrow$ il sostegno della curva è nella striscia orizzontale $-1 \leq x \leq 1$

$y'(t) \geq 0 \Rightarrow y(t)$ monotona crescente $\Rightarrow$ la curva è semplice

$(y'(t) = 0$ solo se $t = 0$ e $x(0) = 0)$

Es. 3 $\quad \gamma \quad \rho = 2\theta \quad \theta = [0, 4\pi] \qquad \int_\gamma \theta \, ds$

$ds = |\vec{r}'(\theta)| \, d\theta \qquad |\vec{r}'(\theta)| = \sqrt{\rho'^2 + \rho^2} = \sqrt{4 + 4\theta^2} = 2\sqrt{1+\theta^2}$

$$\int_\gamma \theta \, ds = \int_0^{4\pi} 2\sqrt{1+\theta^2} \, \theta \, d\theta = \cancel{\frac{2}{2}} \cdot \frac{2}{3} (1+\theta^2)^{3/2} \Big|_0^{4\pi} = \frac{2}{3}\left[\left(1 + (4\pi)^2\right)^{3/2} - 1 \right]$$