SCHEMA SOLUZIONE 1° PARTE ESAME 15.7.2015

Es. 1 $\begin{cases} x = t - \sin t \\ y = 1 - \cos t \end{cases}$ $\begin{cases} x' = 1 - \cos t \\ y' = \sin t \end{cases}$ $t \in [0, \pi/2]$

$$I = \int_\gamma \sqrt{8y - 4y^2}\, ds = 2\int_\gamma \sqrt{2y - y^2}\, ds \qquad ds = |\vec{r}'(t)|\, dt$$

$$|\vec{r}'(t)| = \sqrt{(1-\cos t)^2 + \sin^2 t} = \sqrt{1 + 1 - 2\cos t} = \sqrt{2}\sqrt{1-\cos t}$$

$$\Rightarrow I = 2\sqrt{2}\int_0^{\pi/2} \sqrt{2(1-\cos t) - (1-\cos t)^2}\,\sqrt{1-\cos t}\, dt =$$

$$= 2\sqrt{2}\int_0^{\pi/2} \sqrt{1-\cos t}\,\sqrt{2 - 2\cos t - 1 - \cos^2 t + 2\cos t}\, dt$$

$$= 2\sqrt{2}\int_0^{\pi/2} \sqrt{1-\cos t}\,|\sin t|\, dt = 2\sqrt{2}\int_0^{\pi/2} \sqrt{1-\cos t}\,\sin t\, dt =$$

$$= 2\sqrt{2}\,\frac{2}{3}(1-\cos t)^{3/2}\Big|_0^{\pi/2} = 2\sqrt{2}\,\frac{2}{3} = \frac{4}{3}\sqrt{2}$$

Es. 2 $\underbrace{f(x,y)}$

$$\lim_{(x,y)\to(2,1)} \frac{(x-2)^2(\cos(\pi x) - 1)}{(x-2)^2 + 3x^2(y-1)^2}$$

$$|f(x,y)| \le |\cos(\pi x) - 1| \xrightarrow[x\to 2]{} 0 \quad \Rightarrow \lim_{(x,y)\to(2,1)} f(x,y) = 0$$