

Es. 3

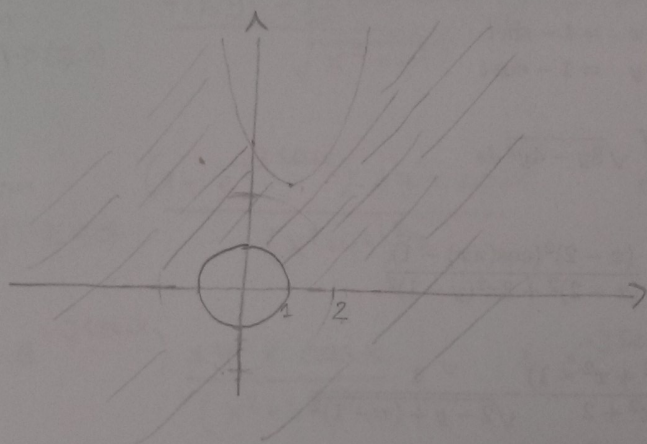
②

$$f(x,y) = \frac{\lg(y^2+x^2-1)}{x^2+2} \quad \frac{1}{\sqrt{2-y+(x-1)^2}}$$

$$\begin{cases} y^2+x^2-1 > 0 \\ 2-y+(x-1)^2 > 0 \end{cases}$$

$$y^2+x^2=1$$

$$y = 2+(x-1)^2$$



$$E = \left\{ (x,y) \in \mathbb{R}^2 : \begin{array}{l} y^2+x^2 > 1 \\ y < 2+(x-1)^2 \end{array} \right\}$$

insieme aperto, illimitato
connesso

Es. 4

$$f(x,y) = \begin{cases} (1 - e^{-3\sin x}) \frac{4y^2 x \cos x}{x^2+y^2} & \text{se } (x,y) \neq (0,0) \\ 0 & \text{se } (x,y) = (0,0) \end{cases}$$

f è definita in tutto \mathbb{R}^2

$$\lim_{(x,y) \rightarrow (0,0)} (1 - e^{-3\sin x}) \frac{4y^2 x \cos x}{x^2+y^2}$$

$$\left| (1 - e^{-3\sin x}) \frac{4y^2 x \cos x}{x^2+y^2} \right| \leq |1 - e^{-3\sin x}| |4x \cos x| \xrightarrow{x \rightarrow 0} 0 \Rightarrow f(x,y) \text{ continua in } (0,0)$$

La funzione è continua in tutto \mathbb{R}^2

$$\frac{\partial f}{\partial x} = 3 \cos x e^{-3\sin x} \frac{4y^2 x \cos x}{x^2+y^2} + (1 - e^{-3\sin x}) \left(4y^2 \frac{\cos x - x \sin x}{x^2+y^2} - \frac{8y^2 x^2 \cos x}{(x^2+y^2)^2} \right)$$

$$\frac{\partial f}{\partial y} = (1 - e^{-3\sin x}) \left[\frac{8y x \cos x}{x^2+y^2} - \frac{8y^3 x \cos x}{(x^2+y^2)^2} \right]$$

Non sono definite in (0,0)

$$\frac{\partial f}{\partial x}(0,0) = \frac{d}{dx} f(x,0) \Big|_{x=0} = 0$$

$$\frac{\partial f}{\partial y}(0,0) = \frac{d}{dy} f(0,y) \Big|_{y=0} = 0$$