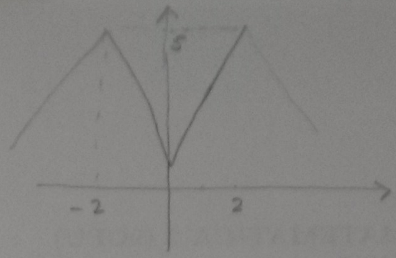
Es. 6 $\quad T=4 \quad$ pari

$f(x) = 2x+1 \quad x \in [0,2]$



$\omega = \frac{2\pi}{T} = \frac{\pi}{2}$

PARI $\Rightarrow b_k = 0$

$$a_0 = \frac{2}{T}\int_0^T f(x)\,dx = 2\,\frac{2}{T}\int_0^{T/2} f(x)\,dx = \int_0^2 (2x+1)\,dx =$$

$$= x^2 + x\Big|_0^2 = 4+2 = 6$$

$$a_k = \frac{2}{T}\int_0^T f(x)\cos(k\omega x)\,dx = 2\cdot\frac{2}{T}\int_0^{T/2} f(x)\cos(k\omega x)\,dx =$$

$$= \int_0^2 (2x+1)\cos(k\omega x)\,dx = \underbrace{+\frac{\operatorname{sen}(k\omega x)(2x+1)}{k\omega}\Big|_0^2}_{=0} - \frac{1}{k\omega}\int_0^2 \operatorname{sen}(k\omega x)\,2\,dx$$

$$= 2\,\frac{1}{(k\omega)^2}\cos(k\omega x)\Big|_0^2 = \frac{2}{(k\omega)^2}\left((-1)^k - 1\right)$$

$$f(x) \sim 3 + \sum_{k=1}^{\infty} \frac{8}{(k\pi)^2}\left((-1)^k - 1\right)\cos\left(k\tfrac{\pi}{2}x\right)$$

La funzione è regolare a tratti e continua $\Rightarrow$ converge puntualmente ad $f(x)$ $\forall x \in \mathbb{R}$

$$\left|\cos\left(k\tfrac{\pi}{2}x\right)\frac{8\left((-1)^k - 1\right)}{(k\pi)^2}\right| \leq \frac{16}{k^2\pi^2} \qquad \sum_{k=1}^{\infty}\frac{16}{k^2\pi^2} < +\infty \Rightarrow \text{la serie converge totalmente}$$

$f'(x)$ non è continua in $[0,T]$ $\Rightarrow$ la serie non è derivabile termine a termine