SCHEMA SOLUZIONE 1 PARTE ESAME 16.9.2015 (1)

Es. 1

$$\begin{cases} x = 2t - \sin 2t \\ y = 1 - \cos 2t \end{cases} \qquad \begin{cases} x' = 2 - 2\cos 2t \\ y' = 2\,\mathrm{sen}\,2t \end{cases} \qquad t \in [0, \pi/4]$$

$$\int_\gamma \sqrt{2y-y^2}\,ds = \int_0^{\pi/4} \left[2(1-\cos 2t) - (1-\cos 2t)^2\right]^{1/2} 2\sqrt{2}\,(1-\cos 2t)^{1/2}\,dt$$

$$|\vec{r}\,'| = \sqrt{2^2(1-\cos 2t)^2 + 2^2(\mathrm{sen}\,2t)^2} = 2\sqrt{1+\cos^2 2t - 2\cos 2t + \mathrm{sen}^2 2t} =$$

$$= 2\sqrt{2(1-\cos 2t)}$$

$$= 2\sqrt{2}\int_0^{\pi/4} (1-\cos 2t)\left(2-(1-\cos 2t)\right)^{1/2} dt$$

$$= 2\sqrt{2}\int_0^{\pi/4} (1-\cos 2t)\sqrt{1+\cos 2t}\,dt =$$

$$= 2\sqrt{2}\int_0^{\pi/4} (1-\cos^2 2t)^{1/2}\sqrt{1-\cos 2t}\,dt =$$

$$= 2\sqrt{2}\int_0^{\pi/4} |\mathrm{sen}\,2t|\,(1-\cos 2t)^{1/2}\,dt =$$

$$= 2\sqrt{2}\int_0^{\pi/2} (1-\cos 2t)^{1/2}\,\mathrm{sen}\,2t\,dt = 2\sqrt{2}\,(1-\cos 2t)^{3/2}\,\frac{2}{3}\,\frac{1}{2}\Big|_0^{\pi/4}$$

$$= \frac{2\sqrt{2}}{3}(1-0) = \frac{2\sqrt{2}}{3}$$