Dato che $\frac{\partial f}{\partial x}$ e $\frac{\partial f}{\partial y}$ sono continue in $\mathbb{R}^2 \setminus \{(0,0)\}$

f è differenziabile in $\mathbb{R}^2 \setminus \{(0,0)\}$.

Studiamo differenziabilità in $(0,0)$

$$\lim_{(x,y)\to(0,0)} \frac{f(x,y) - \frac{\partial f}{\partial x}(0,0)\,x - \frac{\partial f}{\partial y}(0,0)\,y}{\sqrt{x^2+y^2}} =$$

$$= \lim_{(x,y)\to(0,0)} e^{\cos x} \frac{x^2y^2 - 2yx^4}{(x^2+y^2)^{3/2}} = 0 \quad \text{infatti}$$

$$\lim_{x\to 0} e^{\cos x} = e$$

$$\left| \frac{x^2y^2 - 2y\,x^4}{(x^2+y^2)^{3/2}} \right| = \left| \frac{\rho^4 \cos^2\theta \sin^2\theta - 2\rho^5 \sin\theta \cos^4\theta}{\rho^3} \right| \leq \rho + 2\rho^2 \underset{\rho\to 0}{\to} 0$$

$\Rightarrow f$ è differenziabile in $(0,0)$

$$z = f(1,1) + \frac{\partial f}{\partial x}(1,1)(x-1) + \frac{\partial f}{\partial y}(1,1)(y-1) =$$

$$z = e^{\cos 1}\left(-\frac{1}{2}\right) + \left(+\sin 1\, e^{\cos 1} \frac{1}{2} + e^{1}[(-3) + \frac{1}{2}\right](x-1) + e^{\cos 1} \frac{1}{2}(y-1)$$

$$z = e^{\cos 1}\left[-\frac{1}{2} + \left(\frac{1}{2}\sin 1 - \frac{5}{2}\right)(x-1) + \frac{1}{2}(y-1)\right]$$

$$z = \frac{e^{\cos 1}}{2}\left[-1 + (\sin 1 - 5)(x-1) + (y-1)\right]$$