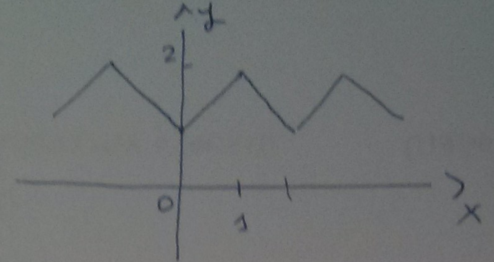
(3)

Es. 6 $\quad f(x) = x+1 \quad x \in [0,1] \quad$ PARI $\quad T=2$



$\omega = \frac{2\pi}{T} = \pi$

funzione pari $\Rightarrow b_k = 0 \quad \forall k = 1, 2, \dots$

$a_0 = \frac{2}{T} \int_0^T f(x)\,dx = 2 \cdot \frac{2}{T} \int_0^{T/2} f(x)\,dx$

$= 2 \int_0^1 (1+x)\,dx = 2\left(1 + \frac{1}{2}\right) = 3$

$a_k = \frac{2}{T} \int_0^T f(x) \cos(k\omega x)\,dx = \frac{2}{T} \cdot 2 \int_0^{T/2} f(x) \cos(k\omega x)\,dx$

$= 2 \int_0^1 (1+x) \cos(k\pi x)\,dx = 2\left[\frac{(1+x)\,\mathrm{sen}(k\pi x)}{k\pi} \Big|_0^1 - \int_0^1 \frac{\mathrm{sen}\, k\pi x}{k\pi}\,dx \right]$

$= 2\left[+ \frac{\cos k\pi x}{(k\pi)^2} \Big|_0^1 \right] = 2\, \frac{(-1)^k - 1}{(k\pi)^2}$

$f(x) \sim \frac{3}{2} + 2 \sum_{k=1}^{\infty} \frac{(-1)^k - 1}{(k\pi)^2} \cos(k\pi x)$

La funzione è regolare a tratti e continua $\Rightarrow$ la serie di Fourier converge puntualmente ad $f(x)$ $\forall x \in [0,T]$.

$\left| \frac{(-1)^k - 1}{(k\pi)^2} \right| \leq \frac{1}{(k\pi)^2} \qquad \sum_k \frac{1}{k^2}$ converge $\Rightarrow$ la serie converge totalmente

$f(0) = f(T)$ ma

la f non è C^1 in $[0,T]$ $\quad f \notin C^1[0,T] \Rightarrow$ la serie non è derivabile termine a termine.