

Es. 4

$$I = \iint_T x^2(y - 2x^3) e^{y+2x^3} dx dy$$

$$T = \{(x,y) \in \mathbb{R}^2 \mid 2x^3 \leq y \leq 3 \quad 2x^3 \geq 1\}$$

$$\begin{cases} u = y - 2x^3 \\ v = y + 2x^3 \end{cases}$$

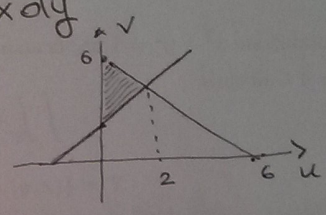
$$\frac{\partial u}{\partial x} = -6x^2 \quad \frac{\partial u}{\partial y} = 1 \quad \begin{pmatrix} -6x^2 & 1 \\ 6x^2 & 1 \end{pmatrix} = -12x^2$$

$$\frac{\partial v}{\partial x} = 6x^2 \quad \frac{\partial v}{\partial y} = 1$$

$$|J| = 12x^2 \Rightarrow du dv = 12x^2 dx dy$$

$$y - 2x^3 \geq 0 \Rightarrow u \geq 0$$

$$\frac{u+v}{2} \leq 3 \quad \frac{v-u}{2} \geq 1$$



$$\begin{aligned} v &\leq 6 - u & v &\geq 2 + u \\ 6 - u &= 2 + u & 4 &= 2u & u &= 2 \end{aligned}$$

$$T' = \{(u,v) : 0 \leq u \leq 2 \quad 2+u \leq v \leq 6-u\}$$

$$I = \frac{1}{12} \int_0^2 du \int_{2+u}^{6-u} dv u e^v = \int_0^2 du u (e^{6-u} - e^{2+u}) =$$

$$= \frac{1}{12} (e^6 \int_0^2 u e^{-u} du - e^2 \int_0^2 u e^u du) =$$

$$= \frac{e^6}{12} \left(-e^{-u} u \Big|_0^2 + \int_0^2 e^{-u} du \right) - \frac{e^2}{12} \left(u e^u \Big|_0^2 - \int_0^2 e^u du \right) =$$

$$= \frac{e^6}{12} (-e^{-2} \cdot 2 - (-1)) - \frac{e^2}{12} (2e^2 - (e^2 - 1)) =$$

$$= \frac{1}{12} (-2e^4 - e^4 + e^6 - 2e^4 + e^4 - e^2) = \frac{-4e^4 + e^6 - e^2}{12}$$