$$U(1,3,2) = 2 + 8 \underbrace{\lg 1}_{=0} - 4 \lg 2 + c = 0$$

$$2 - 4\lg 2 + c = 0 \quad \Rightarrow c = 4\lg 2 - 2$$

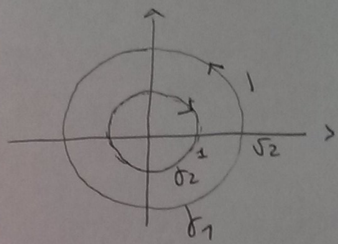
$$U(x,y,z) = x^2 z + 2yz\lg x - y^2 \lg z + 4\lg 2 - 2$$

Es. 6

$$\iint_D y^2 \, dx\, dy$$

Formula di Gauss-Green $\quad \vec{F} = P\hat{i} + Q\hat{j}$

$$\iint_D (Q_x - P_y)\,dx\,dy = \int_{\partial D^+} \vec{F}\cdot d\vec{r}$$




$Q_x = y^2 \quad P = 0 \qquad Q = y^2 x$

$$\int_{\partial D^+} \vec{F}\cdot d\vec{r} = \int_{\gamma_1^+} \vec{F}\cdot d\vec{r} + \int_{\gamma_2^-} \vec{F}\cdot d\vec{r} = \int_{\gamma_1^+} \vec{F}\cdot d\vec{r} - \int_{\gamma_2^+} \vec{F}\cdot d\vec{r} = I_1 - I_2$$

$\gamma_1^+ \Rightarrow \vec{r}_1 \begin{cases} x = \sqrt{2}\cos\theta \\ y = \sqrt{2}\,\text{sen}\,\theta \\ \theta \in [0, 2\pi) \end{cases} \qquad \gamma_2^+\ \vec{r}_2 = \begin{cases} x = \cos\theta \\ y = \text{sen}\,\theta \\ \theta\in[0,2\pi) \end{cases} \qquad \begin{aligned} \vec{r}_1{}' &= (-\sqrt{2}\,\text{sen}\,\theta, \sqrt{2}\cos\theta) \\ \vec{r}_2{}' &= (-\text{sen}\,\theta, \cos\theta) \end{aligned}$

$$I_1 = \int_0^{2\pi} (0, 2\sqrt{2}\,\text{sen}^2\theta\cos\theta)\cdot(-\sqrt{2}\,\text{sen}\,\theta, \sqrt{2}\cos\theta)\,d\theta =$$

$$= \int_0^{2\pi} 4\,\text{sen}^2\theta\cos^2\theta\,d\theta = 4\int_0^{2\pi} (\text{sen}\,2\theta)^2\,d\theta =$$

$$= \int_0^{2\pi} \frac{1-\cos 4\theta}{2}\,d\theta = \pi$$

$$I_2 = \int_0^{2\pi} (0, \text{sen}^2\theta\cos\theta)\cdot(-\text{sen}\,\theta, \cos\theta)\,d\theta = \int_0^{2\pi} \text{sen}^2\theta\cos^2\theta\,d\theta$$

$$= \int_0^{2\pi} \frac{(\text{sen}\,2\theta)^2}{4} = \frac{1}{4}\int_0^{2\pi}\left(\frac{1-\cos 4\theta}{2}\right)d\theta = \frac{\pi}{4} \qquad \Rightarrow I = \frac{3}{4}\pi$$