Es 2 $\quad f(x,y) = (x-1)^2(4y^2-x^2) \quad D = \mathbb{R}^2$ ②

$$\frac{\partial f}{\partial x} = 2(x-1)(4y^2-x^2) - 2x(x-1)^2 = 2(x-1)[4y^2-2x^2+x]$$

$$\frac{\partial f}{\partial y} = 8y(x-1)^2$$

Pti critici

$$\begin{cases} 2(x-1)(4y^2-2x^2+x) = 0 \\ 8y(x-1)^2 = 0 \end{cases}$$

$y = 0 \quad (x-1)(-2x^2+x) = 0 \quad x=1,\ x=0,\ x=\frac{1}{2}$

$x = 1 \quad y = y_0 \quad \Rightarrow$ PTI CRITICI $P_1(0,0)$ $P_2\left(\frac{1}{2},0\right)$ $(1,y_0)$ $y_0 \in \mathbb{R}$

$$\frac{\partial^2 f}{\partial x^2} = 2(4y^2-2x^2+x) + 2(x-1)(-4x+1)$$

$$H_f(1,y_0) = \begin{pmatrix} 2(4y_0^2-1) & 0 \\ 0 & 0 \end{pmatrix}$$

indeterminato

$$\frac{\partial^2 f}{\partial y^2} = 8(x-1)^2 \qquad \frac{\partial^2 f}{\partial x \partial y} = 16(x-1)y$$

$$H_f(0,0) = \begin{pmatrix} -2 & 0 \\ 0 & 8 \end{pmatrix} \Rightarrow P_1(0,0) \text{ pto di sella}$$

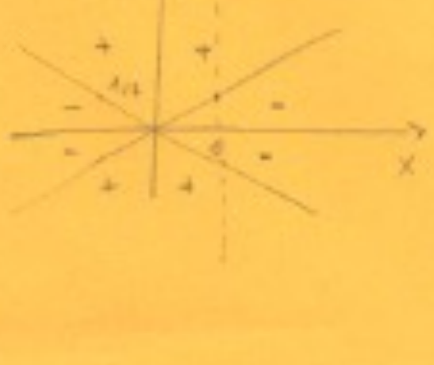
$$H_f(1/2,0) = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \Rightarrow P_2\left(\tfrac{1}{2},0\right) \text{ pto di minimo}$$

Per studiare la natura dei pti $(1,y_0)$ devo studiare l'incremento (il segno)

$$\Delta f(x,y) = f(x,y) - f(1,y_0) = f(x,y) \overset{?}{\geq} 0$$

$f(x,y) = (x-1)^2(4y^2-x^2)$ il segno dipende da $4y^2-x^2 \geq 0 \Leftrightarrow (2y-x)(2y+x) \geq 0$

$y = \frac{1}{2}x \quad y = -1/2\,x$



$-1 < y_0 < 1 \quad (1,y_0)$ pti di massimo $\Delta f < 0$

$y_0 = \pm 1 \quad (1,y_0)$ pti di sella Δf non ha un segno preciso

$y_0 > 1 \quad y_0 < -1 \quad (1,y_0)$ pti di minimo $\Delta f > 0$