

Es. 3

$f(x, y) = y e^{-x}$

$x^2 + 4y^2 = 4$

$g(x, y) = x^2 + 4y^2 - 4$

3

$f(x, y) = x^2 + 4y^2 - 4 \quad \nabla g(x, y) = (2x, 8y) = (0, 0) \Leftrightarrow (x, y) = (0, 0)$

(0,0) μου ειναι οτι κανεις => ειναι η μοναχη μου ησ πτς αυτης!

$$L = y e^{-x} - \lambda (x^2 + 4y^2 - 4) = 0$$

$$\frac{\partial L}{\partial x} = -y e^{-x} - 2\lambda x = 0 \quad -y e^{-x} - 2\lambda x = 0 \Rightarrow 4y^2 = -x \quad (x < 0)$$

$$\frac{\partial L}{\partial y} = e^{-x} - 8\lambda y = 0 \quad e^x = 8\lambda y$$

$$\frac{\partial L}{\partial \lambda} = -(x^2 + 4y^2 - 4) = 0 \quad x^2 - x - 4 = 0 \quad x = \frac{1 \pm \sqrt{1+16}}{2}$$

$$x = \frac{1 + \sqrt{17}}{2} \quad x = \frac{1 - \sqrt{17}}{2} \quad \text{ποσο πιθανοι οσοι ειναι οτι ειναι αρνητικοι}$$

$$y = \pm \frac{1}{2} \sqrt{\frac{\sqrt{17}-1}{2}} \Rightarrow P_2 \left( \frac{1-\sqrt{17}}{2}, -\frac{1}{2} \sqrt{\frac{\sqrt{17}-1}{2}} \right) \quad P_3 \left( \frac{1-\sqrt{17}}{2}, +\frac{1}{2} \sqrt{\frac{\sqrt{17}-1}{2}} \right)$$

$$f \left( \frac{1-\sqrt{17}}{2}, -\frac{1}{2} \sqrt{\frac{\sqrt{17}-1}{2}} \right) = -\frac{1}{2} \sqrt{\frac{\sqrt{17}-1}{2}} e^{\frac{\sqrt{17}-1}{2}}$$

=> P<sub>1</sub> ειναι το σημειο ελαχιστου

P<sub>2</sub> ειναι η μινιμου

P<sub>3</sub> ειναι η μινιμου

$$f \left( \frac{1-\sqrt{17}}{2}, \frac{1}{2} \sqrt{\frac{\sqrt{17}-1}{2}} \right) = \frac{1}{2} \sqrt{\frac{\sqrt{17}-1}{2}} e^{\frac{\sqrt{17}-1}{2}}$$