

$$\text{Es. 4 } \iint_D \lg \sqrt{x^2+y^2} \, dx \, dy$$

$$D = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 9\}$$

Coordinate polari

$$\begin{cases} x = \rho \cos \theta \\ y = \rho \sin \theta \end{cases} \quad \begin{matrix} \theta \in [0, 2\pi] \\ \rho \in [0, 3] \end{matrix}$$

$$\int_0^{2\pi} \int_0^3 \lg \rho \cdot \rho \, d\rho \, d\theta = \int_0^{2\pi} d\theta \int_0^3 \rho \lg \rho \, d\rho$$

$$= 2\pi \left[\frac{1}{2} \rho^2 \lg \rho \right]_0^3 - \int_0^3 \frac{1}{2} \rho^2 \cdot \frac{1}{\rho} \, d\rho = 2\pi \left[\frac{1}{2} 9 \lg 3 - \frac{1}{2} \cdot \frac{1}{2} 9 \right] = 9\pi \left[\lg 3 - \frac{1}{2} \right]$$

$$\lim_{\rho \rightarrow 0} \rho^2 \lg \rho = 0$$