Es. 6 $\qquad \rho = 1+\cos\theta \qquad \theta \in [0, 2\pi] \qquad \begin{cases} x = \rho(\theta)\cos\theta = (1+\cos\theta)\cos\theta \\ y = \rho(\theta)\,\mathrm{sen}\,\theta = (1+\cos\theta)\,\mathrm{sen}\,\theta \end{cases}$

Formula Gauss-Green $\quad \vec{F} = (P, Q)$

$$\iint_D (Q_x - P_y)\,dx\,dy = \int_{\partial D} \vec{F}\cdot d\vec{r}$$

D area racchiusa dalla chiocciola di Pascal

$|D| = \iint_D 1\,dx\,dy \qquad$ scelgo $\vec{F} = (-y, 0) \qquad Q_x - P_y = 1$

a) $\Rightarrow \iint_D dx\,dy \overset{\text{Gauss-Green}}{=} \int_{\partial^+ D} \vec{F}\cdot d\vec{r} = \int_0^{2\pi} \left(-(1+\cos\theta)\,\mathrm{sen}\,\theta, 0\right)\cdot\left(-(1+2\cos\theta)\,\mathrm{sen}\,\theta, \mathrm{sen}^2\theta + (1+\cos\theta)\cos\theta\right) d\theta$

$$\begin{cases} x'(\theta) = -\mathrm{sen}\,\theta\cos\theta + (1+\cos\theta)(-\mathrm{sen}\,\theta) = -(1+2\cos\theta)\,\mathrm{sen}\,\theta \\ y'(\theta) = -\mathrm{sen}^2\theta + (1+\cos\theta)\cos\theta \end{cases}$$

$$= \int_0^{2\pi} (1+3\cos\theta+2\cos^2\theta)\,\mathrm{sen}^2\theta\,d\theta = \int_0^{2\pi} \left(\mathrm{sen}^2\theta + 3\cos\theta\,\mathrm{sen}^2\theta + 2\cos^2\theta\,\mathrm{sen}^2\theta\right) d\theta$$

$\mathrm{sen}^2\theta = \frac{1-\cos 2\theta}{2} \qquad \cos^2\theta = \frac{1+\cos 2\theta}{2} \qquad \mathrm{sen}^2\theta\cos^2\theta = \frac{1-\cos^2 2\theta}{4} = \frac{\mathrm{sen}^2 2\theta}{4} = \frac{1-\cos 4\theta}{8}$

$$= \int_0^{2\pi} \left(\frac{1-\cos 2\theta}{2} + 2\,\frac{1-\cos 4\theta}{8} + 3\cos\theta\,\mathrm{sen}^2\theta\right) d\theta = \frac{3}{4}2\pi = \frac{3}{2}\pi$$

b) $\iint_D dx\,dy \underset{\text{Coordinate polari}}{=} \int_0^{2\pi}\int_0^{1+\cos\theta} \rho\,d\rho\,d\theta = \int_0^{2\pi} \frac{1}{2}(1+\cos\theta)^2\,d\theta =$

$$= \frac{1}{2}\int_0^{2\pi} (1+\cos^2\theta + 2\cos\theta)\,d\theta = \frac{1}{2}\left(2\pi + \int_0^{2\pi} \frac{1+\cos 2\theta}{2}\,d\theta\right) =$$

$$= \frac{1}{2}\left(2\pi + \frac{1}{2}\cdot 2\pi\right) = \frac{2\pi}{2}\cdot\frac{3}{2} = \frac{3}{2}\pi$$