Es. 2

$$f(x,y) = (x+3)^2(y^2-x^2)$$

$$\frac{\partial f}{\partial x} = 2(x+3)(y^2-x^2) - 2x(x+3)^2 = 2(x+3)[y^2-x^2 - x(x+3)] = 2(x+3)[y^2-2x^2-3x]$$

$$\frac{\partial f}{\partial y} = 2y(x+3)^2 = 0$$

$x = -3$ $\quad (-3, y_0)$ retta di pti critici $\quad (0,0) \quad (-\frac{3}{2}, 0)$

$y = 0$ $\quad +x^2 + x(x+3) = 0 \quad x = 0 \quad x + x + 3 = 0 \quad x = -\frac{3}{2}$

$$\frac{\partial^2 f}{\partial x^2} = 2[y^2-2x^2-3x] + 2(x+3)[-4x-3]$$

$$\frac{\partial^2 f}{\partial y^2} = 2(x+3)^2 \qquad \frac{\partial^2 f}{\partial x \partial y} = 4y(x+3)$$

$$Hf(0,0) = \begin{pmatrix} -18 & 0 \\ 0 & 18 \end{pmatrix}$$ $\lambda_1 < 0 \quad \lambda_2 > 0 \Rightarrow$ pto di sella

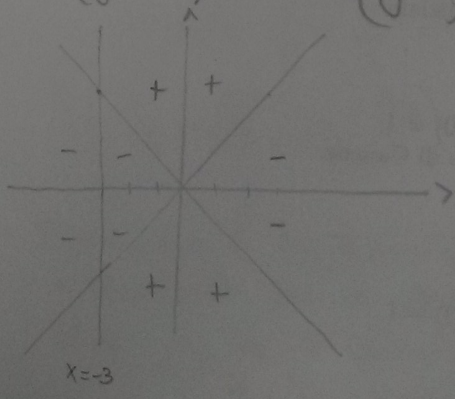
$$Hf(-\tfrac{3}{2},0) = \begin{pmatrix} 9 & 0 \\ 0 & \frac{9}{2} \end{pmatrix}$$ $\lambda_1, \lambda_2 > 0 \Rightarrow$ pto minimo

$$Hf(-3,y_0) = \begin{pmatrix} 2(y_0^2-9) & 0 \\ 0 & 0 \end{pmatrix}$$ indeterminato

$\Rightarrow$ Si deve studiare il segno dell'incremento $\quad f(-3,y_0) = 0$

$\Delta f(x,y) = f(x,y) = (x+3)^2(y^2-x^2) \geq 0 \qquad (x+3)^2 \geq 0$ sempre

$\Rightarrow (y^2-x^2) \geq 0 \qquad (y-x)(y+x) \geq 0$



$\forall$ pti della retta $(-3,y_0)$ sono

$y_0 > 3 \quad y_0 < -3$ dei minimi

$y_0 = 3 \quad y_0 = -3$ dei pti di sella

$-3 < y_0 < 3$ dei pti di max