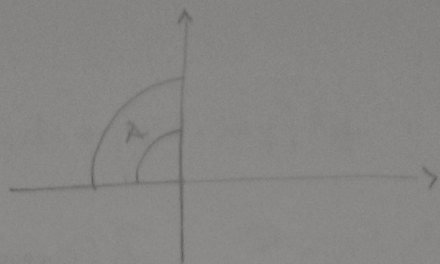


Es. 4



4

$$\text{Area} = \frac{\pi \cdot 4 - \pi}{4} = \frac{3\pi}{4}$$

$$\bar{x} = \frac{1}{A} \int_A x \, dx \, dy = \frac{4}{3\pi} \int_1^2 \int_{\pi/2}^{\pi} \rho \cos \theta \, \rho \, d\rho \, d\theta =$$

$$= \frac{4}{3\pi} \left[ \frac{1}{3} \rho^3 \right]_1^2 \left[ \sin \theta \right]_{\pi/2}^{\pi} = \frac{4}{3\pi} \cdot \frac{1}{3} (8-1) (-1) = -\frac{28}{9\pi}$$

$$\bar{y} = \frac{1}{A} \int_A y \, dx \, dy = \frac{4}{3\pi} \int_1^2 \int_{\pi/2}^{\pi} \rho \sin \theta \, \rho \, d\rho \, d\theta =$$

$$= \frac{4}{3\pi} \left[ \frac{1}{3} \rho^3 \right]_1^2 \left[ -\cos \theta \right]_{\pi/2}^{\pi} = \frac{4}{9\pi} (8-1) (1) = \frac{28}{9\pi}$$

$$\text{Es. 5} \quad \vec{F} = \frac{3y^2 - 3x^2}{x^2 + y^2} \hat{i} - \frac{6xy}{x^2 + y^2} \hat{j}$$

$$\text{Dominio } D = \mathbb{R}^2 \setminus \{0,0\} \quad \vec{r}(t) = (3t^2 + 2, 3 - 2t^2)$$

$$\nabla_x \vec{F} = (\partial_x F_1 - \partial_y F_2) \hat{k}$$

$$\partial_x F_1 = -\frac{6y}{x^2 + y^2} + \frac{12x^2 y}{(x^2 + y^2)^2} \neq \partial_y F_2 = \frac{6y}{x^2 + y^2} - \frac{3y^2 - 3x^2}{(x^2 + y^2)^2} (2y)$$

$\Rightarrow \nabla_x \vec{F} \neq 0$  è campo non è conservativo  $\Rightarrow$  non esiste  
è potenziale  $\downarrow$

$$\int_{\gamma} \vec{F} \cdot d\vec{r} = \int_{\gamma} \vec{F} \cdot \vec{r}' \, dt = \int_0^1 \vec{F}(3t^2 + 2, 3 - 2t^2) \cdot (6t, -4t) \, dt$$

$$\vec{r}'(t) = (6t, -4t)$$