

Es. 6

⑤

$$\oint_{\gamma} (x^3 - x^2y^2) dx + (y^2x - 2xy) dy = \int_{\mathcal{Q}} \vec{F} \cdot d\vec{r}$$

$$\vec{F} = P \hat{i} + Q \hat{j} \quad P = x^3 - x^2y^2 \quad Q = y^2x - 2xy$$

Formula di Gauss-Green

$$\oint_{\mathcal{Q}} \vec{F} \cdot d\vec{r} = \int_{\mathcal{Q}} (Q_x - P_y) dx dy$$

$$\Rightarrow \int_0^1 \int_0^1 (y^2 - 2y + 2x^2y) dx dy =$$

$$= \left. \frac{1}{3} y^3 - y^2 \right|_0^1 + 2 \left(\left. \frac{1}{3} x^3 \right|_0^1 \cdot \left. \frac{1}{2} y^2 \right|_0^1 \right) =$$

$$= \frac{1}{3} - 1 + 2 \left(\frac{1}{3} \cdot \frac{1}{2} \right) = -\frac{2}{3} + \frac{1}{3} = -\frac{1}{3}$$