Es. 1

$$f(x,y) = \begin{cases} \frac{xy^2}{x^2+y^2} & \text{per } (x,y) \neq (0,0) \\ 0 & \text{per } (x,y) = (0,0) \end{cases} \qquad \hat{v} = (\cos\theta, \sin\theta)$$

$$D_v f(0,0) = \frac{d}{dt} f(t\cos\theta, t\sin\theta)\Big|_{t=0} = \cos\theta \sin^2\theta$$

$$g(t) = f(t\cos\theta, t\sin\theta) = \frac{t^3 \cos\theta \sin^2\theta}{t^2} = t\cos\theta\sin^2\theta$$

$$\frac{\partial f}{\partial x}(0,0) = \frac{d}{dx} f(x,0)\Big|_{x=0} = 0$$

$$\frac{\partial f}{\partial y}(0,0) = \frac{d}{dy} f(0,y)\Big|_{y=0} = 0$$

$$\Rightarrow \nabla f(0,0) = (0,0)$$

$\nabla f(0,0) \cdot \hat{v} = 0 \Rightarrow$ la formula del gradiente non è soddisfatta

$$D_v f(0,0) = \cos\theta \sin^2\theta = 0 \qquad \theta = 0, \pi, \pi/2, \frac{3}{2}\pi + 2k\pi$$