

$$f(x, y) = 2x^4 + 2y^4 - 4x^2y$$

$$D = \mathbb{R}^2$$

$$\frac{\partial f}{\partial x} = 8x^3 - 8xy = 8x(x^2 - y) = 0$$

$$\frac{\partial f}{\partial y} = 8y^3 - 4x^2 = 4(2y^3 - x^2) = 0$$

$$x=0 \Rightarrow y=0 \quad y=x^2 \quad 2y^3 - y = 0 \quad y(2y^2 - 1) = 0 \Leftrightarrow y=0 \quad y = \pm \frac{1}{\sqrt{2}}$$

$$(0,0) \quad \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \quad \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$\frac{\partial^2 f}{\partial x^2} = 8(3x^2 - y) \quad \frac{\partial^2 f}{\partial x \partial y} = -8x$$

$$\frac{\partial^2 f}{\partial y^2} = 24y^2$$

$$H_f(0,0) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{Non posso dire nulla}$$

$$H_f\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \begin{pmatrix} 8\left(3\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) & +\frac{8}{\sqrt{2}} \\ \frac{8}{\sqrt{2}} & 24\frac{1}{2} \end{pmatrix} = 8 \begin{pmatrix} \sqrt{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{3}{2} \end{pmatrix}$$

$$\det H_f\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = 8^2 \left(\frac{3}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) = 8^2 \sqrt{2} > 0 \quad \sqrt{2} > 0 \Rightarrow \text{minimo rel.}$$

$$H_f\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = 8 \begin{pmatrix} \sqrt{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{3}{2} \end{pmatrix} \quad \det H_f\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) > 0 \quad \sqrt{2} > 0 \Rightarrow \text{minimo rel.}$$

$g(x, x) = 4x^4 - 4x^3$  che ha un flesso in  $x=0 \Rightarrow (0,0)$  è un pt. di sella