

E 6

$$\oint_C (x^2 - 2x^2y^2)dx + (y^2x - 2x^2y)dy = I$$

La formula di Gauss-Green è data da

$$\vec{F} = P\hat{i} + Q\hat{j} \quad \oint_{\partial D} \vec{F} \cdot d\vec{r} = \iint_D (Q_x - P_y) dx dy$$

$$Q_x = y^2 - 4xy$$

$$P_y = -4x^2y$$

$$I = \int_0^2 dx \int_1^3 dy (y^2 - 4xy + 4x^2y)$$

$$= \int_0^2 dx \left[\frac{1}{3}y^3 - \frac{4xy^2}{2} + \frac{4x^2y^2}{2} \right]_1^3 = \int_0^2 dx \left[(9 - 18x + 18x^2) - \left(\frac{1}{3} - 2x + 2x^2 \right) \right]$$

$$= 9 \int_0^2 (1 - 2x + 2x^2) dx - \int_0^2 dx \left(\frac{1}{3} - 2x + 2x^2 \right)$$

$$= 9 \left(2 - 4 + \frac{2}{3} \cdot 8 \right) - \left(\frac{1}{3} - 4 + \frac{2}{3} \cdot 8 \right) = \frac{144 - 17}{3} + 32 + 18$$

$$= \frac{127}{3} + 50 = \frac{277}{3}$$