Es. 1

$$f(x,y) = \begin{cases} \dfrac{x^2}{y-1} & \text{se } y \neq 1 \\ 0 & \text{se } y = 1 \end{cases}$$

se $y \neq 1$

$\dfrac{\partial f}{\partial x} = \dfrac{2x}{y-1}$ le derivate parziali sono continue in $(1,2)$ $\Rightarrow f$ è differenziabile in $(1,2)$

$\dfrac{\partial f}{\partial y} = -\dfrac{x^2}{(y-1)^2}$ $\quad \dfrac{\partial f}{\partial x}(1,2) = \dfrac{2}{1} = 2 \quad \dfrac{\partial f}{\partial y}(1,2) = -1$

Piano tg $z = 1 + 2(x-1) + (-1)(y-2)$

$z = 2x - y + 1$

$D_v f(0,1) \quad \bar{v} = \cos\theta, \operatorname{sen}\theta$

$g(t) = f(x_0 + tv_1, y_0 + tv_2) = f(t\cos\theta, 1 + t\operatorname{sen}\theta) = \dfrac{t^2\cos^2\theta}{t\operatorname{sen}\theta}$

$g'(t) = \dfrac{\cos^2\theta}{\operatorname{sen}\theta} = D_v f(0,1)$ se $\theta \neq \frac{\pi}{2} + k\pi$ se $\theta = \pi/2 + k\pi$ $D_\theta f(0,1) = \dfrac{\partial f}{\partial y}(0$

$\bar{v} = (0,1)$

$\dfrac{\partial f}{\partial x}(0,1) = \dfrac{d}{dx} f(x,1)\Big|_{x=0} = 0$ $\quad \Rightarrow \quad \nabla f(0,1) = 0$

$\dfrac{\partial f}{\partial y}(0,1) = \dfrac{d}{dy} f(0,y)\Big|_{y=0} = 0$

$\Rightarrow$ se uso la formula del gradiente $D_v f(0,1) = 0$

$\Rightarrow$ La formula del gradiente non è verificata

$\Rightarrow$ la f non è differenziabile in $(0,1)$