

Es. 2 $f(x,y) = (x-2)^2(x^2-9y^2)$

$f \in C^m(\mathbb{R}^2)$

$$\frac{\partial f}{\partial x} = 2(x-2)(x^2-9y^2) + 2x(x-2)^2 = 0$$

$$\frac{\partial f}{\partial y} = -18y(x-2)^2 = 0$$

$$\begin{cases} 2(x-2)^2 [x^2-9y^2 + x(x-2)] = 0 \\ -18y(x-2)^2 = 0 \end{cases}$$

$x=2 \quad y=y_0 \quad y_0 \in \mathbb{R} \quad P_1(0,0) \quad P_2(1,0)$

$y=0 \quad x^2+x^2-2x=0 \quad 2x^2-2x=0 \quad 2x(x-1)=0$

$x=0$

$x=1$

$$\frac{\partial^2 f}{\partial x^2} = 2(x^2-9y^2) + 8x(x-2) + 2(x-2)^2 \quad \frac{\partial^2 f}{\partial x \partial y} = -36y(x-2)$$

$$\frac{\partial^2 f}{\partial y^2} = -18(x-2)^2$$

$H(0,0) = \begin{pmatrix} 8 & 0 \\ 0 & -18 \end{pmatrix} \quad \begin{matrix} \lambda_1 > 0 \\ \lambda_2 < 0 \end{matrix} \Rightarrow P_1 \text{ ptto di sella}$

$H(1,0) = \begin{pmatrix} -4 & 0 \\ 0 & -18 \end{pmatrix} \quad \lambda_1, \lambda_2 < 0 \Rightarrow P_2 \text{ ptto di max}$

$H(2,y_0) = \begin{pmatrix} 2(4-9y_0^2) & 0 \\ 0 & 0 \end{pmatrix}$ dalla matrice hessiana non posso dire nulla (forma indeterminata)

Studio $\Delta f = f(x,y) - f(2,y_0) = f(x,y) \quad f(2,y_0) = 0$

$f(x,y) \geq 0 \Leftrightarrow x^2-9y^2 \geq 0 \Leftrightarrow (x-3y)(x+3y) \geq 0$

$x-3y \geq 0 \Leftrightarrow y \leq \frac{1}{3}x$

$x+3y \geq 0 \Leftrightarrow y \geq -\frac{1}{3}x$