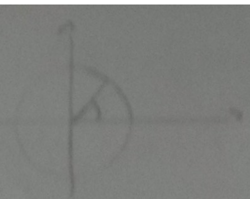
Es. 4



$\frac{8}{3}\pi = 2\pi \qquad R = 3$

$$|\Omega| = \frac{\pi R^2}{6} = \frac{\pi\, 9}{6} = \frac{3}{2}\pi$$

$$\bar{x} = \frac{1}{|\Omega|}\iint_\Omega x\,dx\,dy = \frac{1}{|\Omega|}\int_0^3\int_0^{\pi/3} \rho\cos\theta\,\rho\,d\rho\,d\theta$$

$$= \frac{1}{\frac{3}{2}\pi}\ \frac{1}{3}\rho^3\Big|_0^3\ \sin\theta\Big|_0^{\pi/3} = \frac{2}{3\pi}\ \frac{1}{3}\ 27\ \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{\pi}$$

$$\bar{y} = \frac{1}{|\Omega|}\iint_\Omega y\,dx\,dy = \frac{1}{|\Omega|}\int_0^3\int_0^{\pi/3} \rho\sin\theta\,\rho\,d\rho\,d\theta =$$

$$= \frac{2}{3\pi}\ \frac{1}{3}\rho^3\Big|_0^3 (-\cos\theta)\Big|_0^{\pi/3} = \frac{2}{3\pi}\frac{1}{3}27\left(-\frac{1}{2}+1\right) = \frac{3}{\pi}$$

Es. 5*

Verifichiamo se il campo è irrotazionale $\nabla\wedge\vec{F} = 0$

$$\vec{F} = \frac{x}{(3+x^2+y^2)^{3/2}}\hat{i} + \frac{y}{(3+x^2+y^2)^{3/2}}\hat{j} \qquad D = \mathbb{R}^2 \text{ dominio del campo}$$

$$\nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ F_1 & F_2 & 0 \end{vmatrix} = \hat{k}\left(\partial_x F_2 - \partial_y F_1\right) =$$

$$= \left(\frac{3}{2}\frac{2xy}{(3+x^2+y^2)^{5/2}} - \frac{3}{2}\frac{2xy}{(3+x^2+y^2)^{5/2}}\right)\hat{k} = 0$$

Dato che $D = \mathbb{R}^2$ è semplicemente connesso e $\vec{F}$ è irrotazionale $\Rightarrow$ $\vec{F}$ è conservativo

$$\partial_x U = F_1 = \frac{x}{(3+x^2+y^2)^{3/2}} \Rightarrow U = \int dx\,\frac{x}{(3+x^2+y^2)^{3/2}} = -\frac{1}{(3+x^2+y^2)^{1/2}} + c(y)$$

$$\partial_y U = +\frac{y}{(3+x^2+y^2)^{3/2}} + c'(y) = \frac{y}{(3+x^2+y^2)^{3/2}} \Rightarrow c'(y) = 0 \Rightarrow c(y) = c$$

$$\Rightarrow U(x,y) = -\frac{1}{(3+x^2+y^2)^{1/2}} + c$$

Il campo è conservativo $\Rightarrow \int_C \vec{F}\cdot d\vec{r} = U(P_2) - U(P_1)$

$P_1 = (0,1) \quad P_2(3,-1)$

$$= -\frac{1}{2} + \frac{1}{13} = -\frac{11}{26}$$