COMPITO 1 SCHEMA SOLUZIONE

(1) $y' = (1-y)(3-y)x$ $\quad a(x) = x \quad b(y) = (1-y)(3-y)$ $\quad y=1,\ y=3$ soluzioni costanti

$b \in C^1(\mathbb{R})$, $a \in C(\mathbb{R})$. Il pb di Cauchy ha ~~localmente~~ unicità ~~ext~~ ...

$$\frac{dy}{dx} = (1-y)(3-y)x \quad \text{se } y \neq 1, 3$$

$$\frac{dy}{(1-y)(3-y)} = x\,dx \Rightarrow \int \frac{dy}{(1-y)(3-y)} = \int x\,dx$$

$$\frac{1}{1-y} \cdot \frac{1}{3-y} = \left(\frac{1}{1-y} - \frac{1}{3-y}\right)\frac{1}{2}$$

$$\int \frac{dy}{(1-y)(3-y)} = \frac{1}{2}\int \frac{dy}{1-y} - \frac{1}{2}\int \frac{1}{3-y}\,dy = \frac{1}{2}\int \frac{dy}{y-3} - \frac{1}{2}\int \frac{dy}{y-1} =$$

$$= \frac{1}{2}\lg\left|\frac{y-3}{y-1}\right| = \frac{1}{2}x^2 + c$$

$$\lg\left|\frac{y-3}{y-1}\right| = x^2 + c \qquad \left|\frac{y-3}{y-1}\right| = e^{x^2+c}$$

$$\frac{y-3}{y-1} = c\,e^{x^2} \qquad y-3 = (y-1)c\,e^{x^2}$$

$$y(1 - c\,e^{x^2}) = 3 - c\,e^{x^2}$$

$$y = \frac{3 - c\,e^{x^2}}{1 - c\,e^{x^2}}$$

$$\Rightarrow y = \frac{3 + c\,e^{x^2}}{1 + c\,e^{x^2}} \quad \text{soluzione generale}$$

$$y(0) = 4 \qquad 4 = \frac{3+c}{1+c} \qquad 4 + 4c = 3 + c \qquad 3c = -1 \qquad c = -\frac{1}{3}$$

$$\begin{cases} y(x) = \dfrac{3 - \frac{1}{3}e^{x^2}}{1 - \frac{1}{3}e^{x^2}} = \dfrac{9 - e^{x^2}}{3 - e^{x^2}} \\ y(0) = 4 \end{cases} \qquad \begin{cases} y(x) = \dfrac{9 - e^{x^2}}{3 - e^{x^2}} \\ y(0) = 4 \end{cases}$$

e'