SCHEMA SOLUZIONE ESAME 12.6.2014 1ª PARTE

1) $y' = 2 - 4y^2 \Rightarrow y' = 2(1-2y^2) = 2(1-\sqrt{2}y)(1+\sqrt{2}y)$

Soluzioni costanti $y = \frac{1}{\sqrt{2}}$ $y = -\frac{1}{\sqrt{2}}$

Poniamo $y \neq \pm\frac{1}{\sqrt{2}}$

$$\int \frac{dy}{(1-\sqrt{2}y)(1+\sqrt{2}y)} = \int 2\,dx$$

$$\frac{1}{1-\sqrt{2}y}\cdot\frac{1}{1+\sqrt{2}y} = \frac{1}{2}\left(\frac{1}{1+\sqrt{2}y} + \frac{1}{1-\sqrt{2}y}\right)$$

$$\Rightarrow \frac{1}{2}\int\left(\frac{1}{1+\sqrt{2}y} + \frac{1}{1-\sqrt{2}y}\right)dy = \int 2\,dx$$

$$\frac{1}{\sqrt{2}}\left[\lg|1+\sqrt{2}y| - \lg|1-\sqrt{2}y|\right] = 4x + c$$

$$\lg\left|\frac{1+\sqrt{2}y}{1-\sqrt{2}y}\right| = 4\sqrt{2}x + c\sqrt{2}$$

$$\frac{1+\sqrt{2}y}{1-\sqrt{2}y} = c_1 e^{4\sqrt{2}x} \qquad 1+\sqrt{2}y = (1-\sqrt{2}y)\,c_1 e^{4\sqrt{2}x}$$

$$\sqrt{2}y(1 + c_1 e^{4\sqrt{2}x}) = c_1 e^{4\sqrt{2}x} - 1 \Rightarrow y = \frac{c_1 e^{4\sqrt{2}x} - 1}{\sqrt{2}(1 + c_1 e^{4\sqrt{2}x})}$$

Se y è la velocità la velocità limite

$$v(t) = \lim_{t\to\infty} \frac{c_1 e^{4\sqrt{2}t} - 1}{\sqrt{2}(1+c_1 e^{4\sqrt{2}t})} = \frac{1}{\sqrt{2}}$$