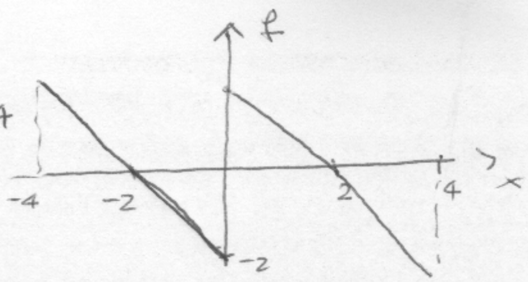
(5)

$f(x) = 2-x \qquad x\in[0,2] \qquad$ dispari $\qquad T=4$



$\omega = \frac{2\pi}{4} = \frac{\pi}{2}$

Funzione dispari $\Rightarrow a_k = 0 \quad k=0,1,2$

$$b_k = \frac{2}{T}\int_0^T f(x)\,\mathrm{sen}(k\omega x)\,dx = 2\cdot\frac{2}{T}\int_0^{T/2}(2-x)\,\mathrm{sen}(k\omega x)\,dx$$

$$= \int_0^2 (2-x)\,\mathrm{sen}(k\omega x)\,dx = 2\left(-\frac{\cos k\omega x}{k\omega}\right)\Big|_0^2 - \int_0^2 x\,\mathrm{sen}(k\omega x)\,dx$$

$$= 2\left(\frac{-(-1)^k}{k\omega} + \frac{1}{k\omega}\right) - \left[-\frac{1}{k\omega}\,x\cos k\omega x\Big|_0^2 + \int_0^2 \frac{\cos k\omega x}{k\omega}\,dx\right] =$$

$$= 2\,\frac{1-(-1)^k}{k\omega} + 2\,\frac{(-1)^k}{k\omega} = \frac{2}{k\omega} = \frac{4}{k\pi}$$

$$f(x) \sim \sum_{k=1}^{\infty} \frac{4}{k\pi}\,\mathrm{sen}\left(k\frac{\pi}{2}x\right) \quad \text{serie di Fourier}$$

La serie di Fourier converge in media quadratica perché $\int_0^T f^2(x)\,dx < \infty$

Uguaglianza di Parseval

$$\int_0^T f^2(x)\,dx = \frac{T}{2}\left[\sum_k (a_k^2+b_k^2) + \frac{a_0^2}{2}\right]$$

$$\int_0^4 \underset{\text{pari}}{f^2(x)}\,dx = 2\int_0^2 (2-x)^2\,dx = 2\left(4x + \frac{1}{3}x^3 - 4\cdot\frac{1}{2}x^2\right)\Big|_0^2 =$$

$$= 2\left(8 + \frac{1}{3}8 - 8\right) = \frac{16}{3}$$

$$\frac{16}{3} = \frac{4}{2}\sum_{k=1}^{\infty}\left(\frac{4}{k\pi}\right)^2 \Rightarrow \frac{1}{6}\pi^2 = \sum_{k=1}^{\infty}\frac{1}{k^2}$$

$f(x)$ regolare a tratti $\Rightarrow$ la serie di Fourier converge $\forall x\in[0,T]$. Converge a $f(x)$ $\forall x\in(0,T)=(0,4)$

Converge a $\frac{f(0)+f(4)}{2} = 0 \quad x=0,T$

$f(0)\neq f(4) \quad f\notin C^1$ La serie NON è derivabile termine a termine