5) $f(x) = 1-x \qquad x\in[0,1]$

f 2-periodica dispari $a_k = 0 \quad k=0,1,2\dots$

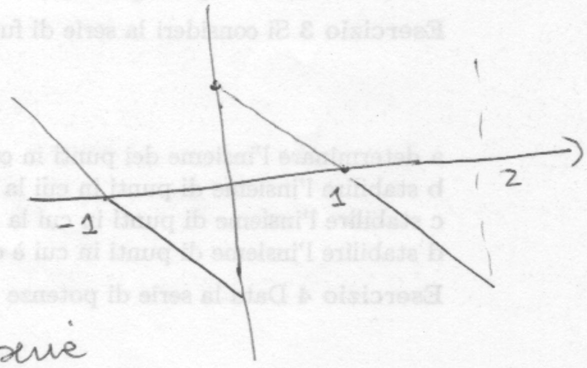
$T=2 \qquad \omega = \frac{2\pi}{T} = \pi$

$$b_k = \frac{2}{T}\int f(x)\,\mathrm{sen}(k\pi x)\,dx = \frac{2}{T}\cdot 2\int_0^1 (1-x)\,\mathrm{sen}(k\pi x)\,dx$$

$$= 2\int_0^1 \mathrm{sen}(k\pi x)\,dx - 2\int_0^1 x\,\mathrm{sen}(k\pi x)\,dx =$$

$$= 2\left[-\frac{\cos(k\pi x)}{k\pi}\Big|_0^1 + x\frac{\cos(k\pi x)}{k\pi}\Big|_0^1 - \int_0^1 \frac{\cos(k\pi x)}{k\pi}\,dx\right]$$

$$= 2\left[-\frac{(-1)^k-1}{k\pi} + \frac{(-1)^k}{k\pi}\right] = \frac{2}{k\pi}$$



$$f(x) \sim \frac{2}{\pi}\sum_{k=1}^{\infty}\frac{\mathrm{sen}(k\pi x)}{k}$$

$f(x)$ è regolare a tratti, la serie (vedi (*)) converge in media quadratica e puntualmente.

Per $x\in(0,2)$ la funzione è continua $\Rightarrow$ la serie converge ad $f(x)$ $\forall x\in(0,2)$.

$x=0 \quad x=2$ la serie converge a $\frac{f(0^+)+f(2^-)}{2} = \frac{1-1}{2} = 0$

(*) $$\int_0^T f^2(x)\,dx = \frac{T}{2}\left[\frac{a_0^2}{2}+\sum_{k=1}^{\infty}(a_k^2+b_k^2)\right] \quad \text{Parseval}$$

$$\int_0^2 (1-x)^2\,dx = \sum_{k=1}^{\infty} b_k^2 \iff 2+\frac{8}{3}-2\frac{1}{2}\cdot 4 = \sum_{k=1}^{\infty}\frac{4}{\pi^2k^2}$$

$$\frac{2}{3} = \sum_{k=1}^{\infty}\frac{4}{\pi^2k^2} \Rightarrow \frac{\pi^2}{6} = \sum_{k=1}^{\infty}\frac{1}{k^2}$$

La serie non è derivabile termine a termine $f(0^+)\neq f(2^-)$