

$$\textcircled{1} \quad y' = -\frac{2x}{2+x^2}y + \frac{1}{x(2+x^2)} \Rightarrow y' = a(x)y + b(x)$$

Eq. linear del 1° ordine non omogenea

$$y(x) = e^{\int a(x)dx} \left(c + \int b(x) e^{-\int a(x)dx} dx \right) \quad c \in \mathbb{R}$$

$$a(x) = -\frac{2x}{2+x^2} \quad b(x) = \frac{1}{x(2+x^2)}$$

$$\int a(x)dx = -\int \frac{2x}{2+x^2} dx = -\lg(2+x^2)$$

$$\Rightarrow y(x) = \frac{1}{2+x^2} \left(c + \int \frac{1}{x(2+x^2)} dx \right) = \frac{1}{2+x^2} (c + \lg|x|) \quad x \neq 0$$

$$y(-2) = 0$$

$$y(-2) = \frac{1}{6} (c + \lg 2) = 0 \quad c = -\lg 2$$

$$y(x) = \frac{1}{2+x^2} (-\lg 2 + \lg|x|) \quad \text{è definita su } (-\infty, 0)$$

$$\Rightarrow y(x) = \frac{1}{2+x^2} (-\lg 2 + \lg(-x)) = \frac{1}{2+x^2} \lg\left(-\frac{x}{2}\right)$$