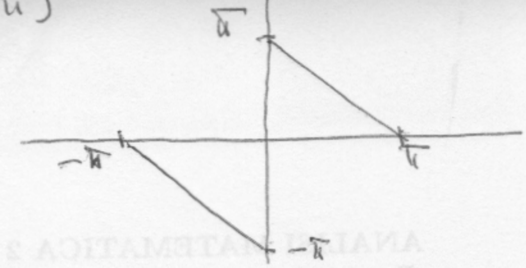
(5) $T = 2\pi \quad f(x) = \pi - x \quad x \in [0, \pi)$



dispari

$a_k = 0$

$$b_k = \frac{2}{\pi} \int_0^{\pi} (\pi - x)\, \mathrm{sen}(kx)\, dx \qquad k \geq 1$$

$$= \frac{2}{\pi} \left[-\pi \frac{1}{k} \cos kx \Big|_0^{\pi} - \int_0^{\pi} x\, \mathrm{sen}\, kx\, dx \right] =$$

$$= \frac{2}{\pi} \left[\frac{\pi}{k} (1 - (-1)^k) - \left(-\frac{x}{k} \cos kx \Big|_0^{\pi} + \int_0^{\pi} \frac{\cos kx}{k} dx \right) \right] =$$

$$= \frac{2}{\pi} \left[\frac{\pi}{k} (1 - (-1)^k) + \frac{\pi}{k} (-1)^k + 0 \right] = \frac{2}{k}$$

$$f(x) \sim \sum_{k=1}^{\infty} \frac{2}{k} \mathrm{sen}(kx)$$

La serie converge in media quadratica (serie di Fourier) perché $\int_0^{2\pi} f^2 dx < +\infty$

Uguaglianza di Parseval

$$\int_0^{2\pi} f^2(x) dx = \pi \left[\frac{a_0^2}{2} + \sum_k (a_k^2 + b_k^2) \right]$$

$$\int_0^{2\pi} f^2(x) dx = \int_{-\pi}^{0} (-\pi - x)^2 dx + \int_0^{\pi} (\pi - x)^2 dx =$$

$$= \int_{-\pi}^{0} (\pi^2 + x^2 + 2\pi x) dx + \int_0^{\pi} (\pi^2 + x^2 - 2\pi x) dx =$$

$$= \pi^2 \pi + \frac{1}{3} x^3 \Big|_{-\pi}^{0} + 2\pi \frac{1}{2} x^2 \Big|_{-\pi}^{0} + \pi^2 \pi + \frac{1}{3} x^3 \Big|_0^{\pi} - 2\pi \frac{1}{2} x^2 \Big|_0^{\pi} =$$

$$= \pi^3 + \frac{1}{3}\pi^3 - \pi^3 + \pi^3 + \frac{1}{3}\pi^3 - \pi^3 = \frac{2}{3}\pi^3$$

oppure se $f(x)$ è dispari $\Rightarrow f^2(x)$ è pari

$$\Rightarrow \int_0^{2\pi} f^2(x) dx = \int_{-\pi}^{\pi} f^2(x) dx = 2 \int_0^{\pi} f^2(x) dx = 2 \int_0^{\pi} (\pi - x)^2 dx$$

$$= 2 \left(\pi^2 \pi + \frac{1}{3}\pi^3 - 2\pi \frac{1}{2} \pi^2 \right) = \frac{2}{3}\pi^3$$

$$\frac{2\pi^3}{3} = \pi \sum_k \frac{4}{k^2} \Rightarrow \sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}$$