④ $\displaystyle\sum_{n=0}^{\infty} \frac{2n+1}{n^2+3} (2x)^n$ Conv. totale $\forall\ r < 1/2$, $x \in [-r, r]$ ④

$a_n = \dfrac{2^n (2n+1)}{n^2+3}$ $\dfrac{a_{n+1}}{a_n} = \dfrac{2^{\cancel{n}+1}(2(n+1)+1)}{(n+1)^2+3} \dfrac{n^2+3}{\cancel{2^n}(2n+1)} \xrightarrow[n\to\infty]{} 2$

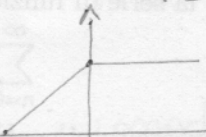
$R = \frac{1}{2}$ raggio di convergenza

~~[illegible]~~ $x = \frac{1}{2} \rightarrow \displaystyle\sum_{n=0}^{\infty} \frac{2n+1}{n^2+3}$ non converge (diverge)

$x = -\frac{1}{2} \rightarrow \displaystyle\sum_{n=0}^{\infty} \frac{2n+1}{n^2+3} (-1)^n$ Converge Criterio di Leibnitz

⑤ $f(x) = \begin{cases} 2 & \text{se } x \in [0,2] \\ 2+x & \text{se } x \in [-2,0) \end{cases}$ $\omega = \frac{2\pi}{4} = \frac{\pi}{2}$

$T = 4$



$a_k = \dfrac{2}{T} \displaystyle\int_0^T f(x) \cos(k\omega x)\,dx = \frac{1}{2}$ =

$= \dfrac{1}{2} \displaystyle\int_{-2}^{2} f(x) \cos(kx\omega)\,dx$

$a_0 = \dfrac{1}{2}\left[\displaystyle\int_{-2}^{0} (x+2)\,dx + \int_0^2 2\,dx\right] = \frac{1}{2}\left[\frac{1}{2}x^2 + 2x\Big|_{-2}^{0} + 2x\Big|_0^2\right] =$

$= \dfrac{1}{2}\left[-\frac{1}{2}\cdot 4 + 4 + 4\right] = 3$

$a_k = \dfrac{1}{2}\left[\displaystyle\int_{-2}^{0} (x+2)\cos(k\omega x)\,dx + \int_0^2 2\cos(k\omega x)\,dx\right] =$ $a_k = \dfrac{1}{2k^2\omega^2}(1-(-1)^k)$

$= \dfrac{1}{2}\left[\displaystyle\int_{-2}^{0} x\cos(k\omega x)\,dx + 2\int_0^2 \cos(k\omega x)\,dx\right] =$

$= \dfrac{1}{2}\left[\dfrac{x}{k\omega}\operatorname{sen}(k\omega x)\Big|_{-2}^{0} - \displaystyle\int_{-2}^{0} \frac{\operatorname{sen}(k\omega x)}{k\omega}\,dx + 2\int_0^2 \cos(k\omega x)\,dx\right] =$

$= \dfrac{1}{2}\left[\underbrace{-\dfrac{2}{k\omega}\operatorname{sen}\left(-\dfrac{k\pi}{2}2\right)}_{=0} + \dfrac{1}{(k\omega)^2}\cos(k\omega x)\Big|_{-2}^{0} + \underbrace{2\dfrac{1}{k\omega}\operatorname{sen}(k\omega x)\Big|_0^2}_{=0}\right]$