$$b_k = \frac{1}{2}\int_{-2}^{2} f(x)\,\mathrm{sen}\left(k\frac{\pi}{2}x\right)dx = \frac{1}{2}\int_{-2}^{0}(2+x)\,\mathrm{sen}\left(k\frac{\pi}{2}x\right)dx$$

$$+\frac{1}{2}\int_{0}^{2} 2\,\mathrm{sen}\left(k\frac{\pi}{2}x\right)dx = \frac{1}{2}\cdot 2\underbrace{\int_{-2}^{2}\mathrm{sen}\left(k\frac{\pi}{2}x\right)dx}_{=0} +$$

$$+\frac{1}{2}\int_{-2}^{0} x\,\mathrm{sen}\left(k\frac{\pi}{2}x\right)dx = \frac{1}{2}\left[\frac{x}{k\frac{\pi}{2}}\left(-\cos\left(k\frac{\pi}{2}x\right)\right)\Big|_{-2}^{0} + \frac{1}{k\pi/2}\underbrace{\int_{-2}^{0}\cos\left(\frac{k\pi}{2}x\right)dx}_{=0}\right]$$

$$= -\frac{1}{2}\,\frac{2}{k\pi/2}(-1)^k = -\frac{2}{k\pi}(-1)^k$$

$$f(x) \sim \frac{3}{2} + \sum_{k=1}^{\infty}\left[\frac{1-(-1)^k}{2k^2\omega^2}\cos(k\omega x) + \frac{(-1)^{k+1}}{k\omega}\,\mathrm{sen}(k\omega x)\right]$$

(b) La serie di Fourier converge sempre in media quadratica

$$\int_{-2}^{2} f^2(x)\,dx = \frac{T}{2}\left[\frac{a_0^2}{2} + \sum_{k=1}^{\infty}\left[\frac{(1-(-1)^k)^2}{4k^4\omega^4} + \frac{1}{k^2\omega^2}\right]\right]$$

$$\int_{-2}^{0}(2+x)^2\,dx + \int_{0}^{2} 4\,dx = \int_{0}^{2} y^2\,dy + 4\cdot 2 =$$

$$= \frac{1}{3}\cdot 8 + 8 = \frac{32}{3}$$

La funzione è regolare a tratti

$$\frac{32}{3}\cdot\frac{1}{2} - \frac{9}{2} = \sum_{k=1}^{\infty}\left[\frac{(1-(-1)^k)^2}{4k^4\omega^4} + \frac{1}{k^2\omega^2}\right]$$

$$\frac{5}{6} = \sum_{k=1}^{\infty}\left[\frac{(1-(-1)^k)^2}{4k^4\omega^4} + \frac{1}{k^2\omega^2}\right]$$

La serie di Fourier converge puntualmente ad $f(x)$ $\forall x \in (-2,2)$

in $x=-2,2$ converge a $\frac{f(-2)+f(2)}{2} = 1$

La serie delle derivate prime non converge $f(-2) \neq f(2)$