

$$1) \quad y' = \left(\frac{4}{5} - y\right)(2 - y)x$$

Sol. costanti: $y = 4/5$ $y = 2$

$$\frac{y'}{(y - 4/5)(y - 2)} = x \quad \frac{1}{(y - 4/5)(y - 2)} = \left(\frac{1}{y - 2} - \frac{1}{y - 4/5}\right) \frac{5}{6}$$

$$-4/5 + 2 =$$

$$\frac{5}{6} \int \left(\frac{dy}{y - 2} - \int \frac{dy}{y - 4/5} \right) = \int x dx$$

$$\frac{5}{6} \left[\lg|y - 2| - \lg|y - 4/5| \right] = \frac{1}{2} x^2 + c$$

$$\lg \left| \frac{y - 2}{y - 4/5} \right| = \frac{3}{5} x^2 + c_1$$

$$\frac{y - 2}{y - 4/5} = c_2 e^{\frac{3}{5} x^2}$$

$$y - 2 = (y - 4/5) c_2 e^{\frac{3}{5} x^2}$$

$$y(1 - c_2 e^{\frac{3}{5} x^2}) = 2 - 4/5 c_2 e^{\frac{3}{5} x^2}$$

$$y = \frac{2 - 4/5 c_2 e^{\frac{3}{5} x^2}}{1 - c_2 e^{\frac{3}{5} x^2}} = \frac{2}{5} \frac{5 - 2 c_2 e^{\frac{3}{5} x^2}}{1 - c_2 e^{\frac{3}{5} x^2}}$$

$$y(0) = 1$$

$$\frac{1 - 2}{1 - 4/5} = c_2 \Rightarrow \frac{-1}{1/5} = -5 = c_2$$

$$y = \frac{2}{5} \frac{5 + 10 c_2 e^{\frac{3}{5} x^2}}{1 + 5 e^{\frac{3}{5} x^2}} = \frac{2(1 + 2 c_2 e^{\frac{3}{5} x^2})}{1 + 5 e^{\frac{3}{5} x^2}}$$

definita $\forall x \in \mathbb{R}$