) $y'' + 4y' + 3y = \cos(2x)$

omogenea associata $z'' + 4z' + 3z = 0$

$\lambda^2 + 4\lambda + 3 = 0$

$\lambda = -2 \pm \sqrt{4-3} = -2 \pm 1 = \begin{cases} -1 \\ -3 \end{cases}$

$z(x) = c_1 e^{-3x} + c_2 e^{-x}$

Studiamo la sol. particolare di

$y'' + 4y' + 3y = e^{i2x}$

$\tilde{y}(x) = A e^{i2x}$ $\quad \tilde{y}' = 2iA e^{i2x}$ $\quad \tilde{y}'' = -4A e^{i2x}$

$(-4 + 8i + 3) A e^{i2x} = e^{i2x}$ $\quad A = \dfrac{1}{-1+8i}$

$\tilde{y}(x) = \dfrac{-1-8i}{65}\left(\cos(2x) + i \operatorname{sen}(2x)\right)$

La sol. particolare dell'eq. di partenza è

$y(x) = \operatorname{Re}(\tilde{y}(x)) = -\dfrac{1}{65}\cos(2x) + \dfrac{8}{65}\operatorname{sen} 2x$

$y(x) = c_1 e^{-3x} + c_2 e^{-x} - \dfrac{1}{65}\cos(2x) + \dfrac{8}{65}\operatorname{sen}(2x)$ int. generale

$y(0) = 0 = c_1 + c_2 - \dfrac{1}{65}$ $\quad c_1 = \dfrac{1}{65} - c_2$

$y'(0) = 1 = -3c_1 - c_2 + \dfrac{16}{65}$ $\quad -3\left(\dfrac{1}{65} - c_2\right) - c_2 + \dfrac{16}{65} = 1$

$2c_2 = 1 - \dfrac{16}{65} + \dfrac{3}{65} = \dfrac{65-13}{65} = \dfrac{52}{65}$ $\quad c_2 = \dfrac{26}{65}$

$c_1 = \dfrac{1}{65} - \dfrac{26}{65} = -\dfrac{25}{65} = -\dfrac{5}{13}$

$y(x) = -\dfrac{5}{13} e^{-3x} + \dfrac{26}{65} e^{-x} - \dfrac{1}{65}\cos(2x) + \dfrac{8}{65}\operatorname{sen} 2x$