COMPITO A 2° PARZIALE 28.5.2014 Schema di soluzione. (1)

(1)
$$\rho=\left(\cos\frac{\theta}{2}\right)^2 \qquad \theta\in[0,2\pi]$$

$$\vec{r}=\rho(\cos\theta,\operatorname{sen}\theta) \qquad \rho'=-2\cos\frac{\theta}{2}\operatorname{sen}\frac{\theta}{2}\,\frac{1}{2}$$

$$\vec{r}\,'=\rho'(\cos\theta,\operatorname{sen}\theta)+\rho(-\operatorname{sen}\theta,\cos\theta)$$

$$|\vec{r}\,'|=\left(\rho'^2+\rho^2\right)^{1/2}=\left(\cos^2\frac{\theta}{2}\operatorname{sen}^2\frac{\theta}{2}+\cos^4\frac{\theta}{2}\right)^{1/2}=\left(\cos^2\frac{\theta}{2}\right)^{1/2}$$

$$=\left|\cos\frac{\theta}{2}\right|$$

$$l(\gamma)=\int_0^{2\pi}\left|\cos\frac{\theta}{2}\right|d\theta \qquad \alpha=\frac{\theta}{2} \qquad d\alpha=\frac{d\theta}{2}$$

$$=\int_0^{\pi}|\cos\alpha|\,2\,d\alpha=2\left[\int_0^{\pi/2}\cos\alpha\,d\alpha+\int_{\pi/2}^{\pi}-\cos\alpha\,d\alpha\right]$$

$$=2\left[+\operatorname{sen}\alpha\Big|_0^{\pi/2}-\operatorname{sen}\alpha\Big|_{\pi/2}^{\pi}\right]=2(1+1)=4$$

(2)
$$\lim_{(x,y)\to(0,0)}\frac{x^5+xy^2+4x^2y}{y^2+2x^4+3y^4} \quad \text{non esiste!}$$

$$\left|\frac{x^5+xy^2+4x^2y}{y^2+2x^4+3y^4}\right|\le\left|\frac{x^5}{y^2+2x^4+3y^4}+\frac{xy^2}{y^2+2x^4+3y^4}+\frac{4x^2y}{y^2+2x^4+3y^4}\right| \quad (*)$$

$$\le\left|\frac{x^5}{2x^4}\right|+\frac{|x|y^2}{y^2}+\frac{4x^2|y|}{y^2+2x^4+3y^4}\le|x|+|x|+\frac{4x^2|y|}{y^2+2x^4+3y^4}$$

I primi due termini tendono a zero ma il terzo no.
Vediamo in dettaglio il terzo termine in $(*)$

$$g(x,y)=\frac{4x^2y}{y^2+2x^4+3y^4} \qquad g(x,x^2)=\frac{4x^4}{3x^4+3x^8}\xrightarrow[x\to0]{}\frac{4}{3}$$

$$g(x,-x^2)=\frac{-4x^4}{3x^4+3x^8}\xrightarrow[x\to0]{}-\frac{4}{3}$$

Dunque il limite non esiste.