SCHEMA SOLUZIONE ESAME 15.7.2014 2° PARTE

① $\gamma: \quad y = \lg x \quad x \in [1/2, 2]$

$$\vec{r}(x) = (x, \lg x)$$

$$\vec{r}\,'(x) = \left(1, \frac{1}{x}\right)$$

$$I = \int_\gamma x \, ds = \int_{1/2}^{2} x \sqrt{1+\left(\frac{1}{x}\right)^2} \, dx =$$

$$= \int_{1/2}^{2} \sqrt{1+x^2} \, dx = \int_{t_1}^{t_2} (\cosh t)^2 dt = \int_{t_1}^{t_2} \frac{e^{2t}+e^{-2t}-2}{4} dt$$

$x = \operatorname{senh} t \quad dx = \cosh t \, dt$

$t_1: \ \frac{1}{2} = \operatorname{senh} t_1 \qquad t_2: \ 2 = \operatorname{senh} t_2$

$\frac{1}{2} = \frac{e^{t_1}-e^{-t_1}}{2} \qquad 2 = \frac{e^{t_2}-e^{-t_2}}{2}$

$\frac{1}{4}+1 = \left(\frac{e^{t_1}+e^{-t_1}}{2}\right)^2 \qquad 4+1 = \left(\frac{e^{t_2}-e^{-t_2}}{2}\right)^2$

$\frac{\sqrt{5}}{2} = \frac{e^{t_1}+e^{-t_2}}{2} \qquad \sqrt{5} = \frac{e^{t_2}-e^{-t_2}}{2}$

$\frac{1+\sqrt{5}}{2} = e^{t_1} \quad t_1 = \lg \frac{1+\sqrt{5}}{2} \qquad 2+\sqrt{5} = e^{t_2} \quad t_2 = \lg(2+\sqrt{5})$

$$\Rightarrow I = \frac{1}{4}\left[\frac{1}{2}e^{+2t} - \frac{1}{2}e^{-2t} - 2t\right]_{t_1}^{t_2} = \frac{1}{4}\left[\operatorname{senh} t - 2t\right]_{t_1}^{t_2}$$

$$= \frac{1}{4}\left[2 - \frac{1}{2} - 2\left(\lg 2+\sqrt{5} - \lg\frac{(1+\sqrt{5})}{2}\right)\right] = \frac{1}{4}\left[\frac{3}{2} - 2\lg\frac{(2+\sqrt{5})2}{1+\sqrt{5}}\right]$$

Quindi

$$\bar{x} = \frac{1}{m}\int_\gamma x \, ds = \frac{1}{L(\gamma)}\int_{1/2}^{2} x(t)|\vec{r}\,'(t)| \, dt = \frac{I}{L(\gamma)}$$