(2)

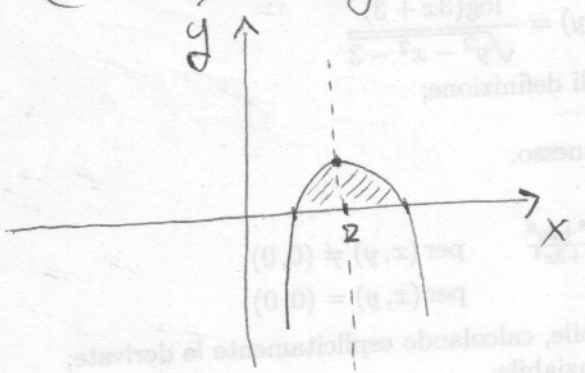
$$\lim_{(x,y)\to(0,0)} (e^{xy}-1)\,\frac{3y^2+x^3\cos x}{y^2+x^2}$$

$$\left|(e^{xy}-1)\,\frac{3y^2+x^3\cos x}{y^2+x^2}\right| \le |e^{xy}-1|\,(3+|x\cos x|) \xrightarrow[(x,y)\to(0,0)]{} 0$$

il limite è 0.

(3) $$f(x,y)=\frac{\lg(2y)}{x-2}\sqrt{1-y-(x-2)^2}$$

$$\begin{cases} 2y>0 \Rightarrow y>0 \\ x-2\neq 0 \Rightarrow x\neq 2 \\ 1-y-(x-2)^2\geq 0 \Rightarrow y\leq 1-(x-2)^2 \end{cases}$$



Insieme né aperto né chiuso illimitato, non connesso

(4) $$f(x,y)=\begin{cases}\dfrac{(x+y)\,\sin(x^2+y^2)}{x^2+y^2} & (x,y)\neq(0,0) \\ 0 & (x,y)=(0,0)\end{cases}$$

$$\lim_{(x,y)\to(0,0)} \frac{(x+y)\,\sin(x^2+y^2)}{x^2+y^2}=0 \Rightarrow f \text{ è continua in } (0,0)$$

infatti $$\left|(x+y)\,\frac{\sin(x^2+y^2)}{x^2+y^2}\right| = \left|\rho(\cos\vartheta+\sin\vartheta)\,\frac{\sin\rho^2}{\rho^2}\right| \le 2\rho\,\frac{\sin\rho^2}{\rho^2} \xrightarrow[\rho\to 0]{} 0$$

$\vec{v}=(\cos\vartheta,\sin\vartheta)$ $\vec{v}=t(\cos\vartheta,\sin\vartheta)$

$$f(0+t\vec{v})=\frac{t(\cos\vartheta+\sin\vartheta)\sin t^2}{t^2}=g(t) \qquad D_{\vec{v}}f(0,0)=g'(0)$$

$$g'(t)=(\cos\vartheta+\sin\vartheta)\,\frac{d}{dt}\,\frac{\sin t^2}{t}=(\cos\vartheta+\sin\vartheta)\left[\frac{\cos t^2\cdot 2t}{t}-\frac{1}{t^2}\sin t^2\right]$$

$$g'(0)=(\cos\vartheta+\sin\vartheta) \qquad f_x(0,0)=1 \quad \vartheta=0 \qquad f_y(0,0)=+1 \quad \vartheta=+1$$