(6) $f(x,y) = 3y e^{-2(x-1)} + (2x-1)y^2 + 2e^{x-1}$

$f(1,y)=0 \iff 3y + y^2 + 2 = 0 \qquad y^2+3y+2=0$

$$y = \frac{-3\pm\sqrt{9-8}}{2} = \frac{-3\pm 1}{2} = \begin{cases} -2 \\ -1 \end{cases}$$

$f_x = 3y\, e^{-2(x-1)}(-2) + 2y^2 + 2e^{x-1}$

$f_y = 3e^{-2(x-1)} + 2y(2x-1)$

$f_y(1,-1) = 3-2 = 1 \neq 0$

$f \in C^1(\mathbb{R})$

Posso applicare il teorema di Dini per la funzione implicita nel pto $(1,-1)$

$$g'(1) = -\frac{f_x(1,-1)}{f_y(1,-1)} = -\frac{6+2+2}{1} = -10$$

(7)

a) $$\int_0^{\pi/2}\int_r^R \rho\cos\theta\, e^{\rho\sin\theta}\rho\, d\rho\, d\theta = \int_r^R \rho^2\left(\int_0^{\pi/2}\cos\theta\, e^{\rho\sin\theta}\, d\theta\right) =$$

$$= \int_r^R\left(\rho^2\ \frac{1}{\rho}\, e^{\rho\sin\theta}\Big|_0^{\pi/2}\right)d\rho = \int_r^R \rho\,(e^\rho - 1)\, d\rho$$

$$= \int_r^R \rho e^\rho\, d\rho - \frac{1}{2}(R^2-r^2) = \rho e^\rho\Big|_r^R - \int_r^R e^\rho\, d\rho - \frac{1}{2}(R^2-r^2) =$$

$$= Re^R - re^r - (e^R - e^r) - \frac{1}{2}(R^2-r^2) = e^R(R-1) + e^r(1-r) - \frac{1}{2}(R^2-r^2)$$

b) $$\int_0^1 dx\int_0^{2x} dy\ x e^y = \int_0^1 x\left[\left(e^y\right)\Big|_0^{2x}\right]dx = \int_0^1 x\,(e^{2x}-1)\,dx$$

$$= \frac{1}{2}x e^{2x}\Big|_0^1 - \frac{1}{2}\int_0^1 e^{2x} - \frac{1}{2}x^2\Big|_0^1 = \frac{1}{2}e^2 - \frac{1}{4}(e^2-1) - \frac{1}{2}$$

$$= \frac{1}{4}(e^2-1)$$