SCHEMA SOL. 16.9.2014 2° PARTE

Es. 1 $\displaystyle\int_\gamma \sqrt{y}\, ds$

$$\begin{cases} x = t\cos t \\ y = t^2 \\ z = t \operatorname{sen} t \end{cases} \qquad ds = |r'(t)|\,dt \qquad t \in [0, 2\pi]$$

$$r'(t) = \left(\cos t - t \operatorname{sen} t,\ 2t,\ \operatorname{sen} t + t\cos t \right)$$

$$|r'(t)| = \sqrt{(\cos t - t\operatorname{sen} t)^2 + 4t^2 + (\operatorname{sen} t + t\cos t)}$$

$$= \sqrt{1 + 5t^2}$$

$$\int_\gamma \sqrt{y}\, ds = \int_0^{2\pi} t\sqrt{1+5t^2}\, dt = \frac{2}{3}\left(1+5t^2\right)^{3/2} \frac{1}{10}\Big|_0^{2\pi}$$

$$= \frac{1}{15}\left(\left(1 + 5\cdot 4\pi^2\right)^{3/2} - 1 \right) = \frac{1}{15}\left(\left(1+20\pi^2\right)^{3/2} - 1 \right)$$

Es. 2 $$\lim_{(x,y)\to(0,0)} \underbrace{\left(e^{x+y} - 1\right) \frac{2y^2\cos y + x^2 \operatorname{sen} x}{y^2 + x^2}}_{f(x,y)} = \lim_{(x,y)\to(0,0)} \tilde{f}(x,y)\, g(x,y)$$

$$\lim_{(x,y)\to(0,0)} \tilde{f}(x,y) = 0$$

$$|g(x,y)| \le 2 + 1 = 3$$

$$\Rightarrow \left|\tilde{f}(x,y)\, g(x,y)\right| \le 3\left|\tilde{f}(x,y)\right| \xrightarrow[(x,y)\to(0,0)]{} 0$$

$$\Rightarrow \lim_{(x,y)\to(0,0)} f(x,y) = 0$$