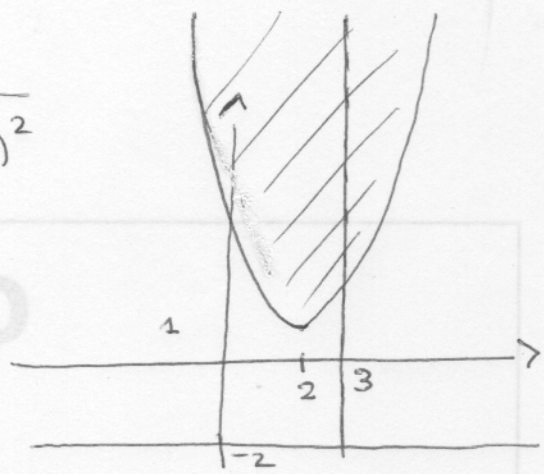
Es. 3

$$f(x,y) = \frac{\lg(2+y)}{x-3} \, \frac{1}{\sqrt{y-1-(x-2)^2}}$$



$$\begin{cases} x \neq 3 \\ 2+y > 0 \\ y-1-(x-2)^2 > 0 \end{cases} \qquad \begin{cases} x \neq 3 \\ y > -2 \\ y > 1+(x-2)^2 \end{cases}$$

$$E = \left\{ (x,y) \in \mathbb{R}^2 : \; x \neq 3 \;\; y > 1+(x-2)^2 \right\}$$

Aperto, illimitato, non connesso.

Es. 4

$$f(x,y) = \begin{cases} (x+y)\,\dfrac{\cos(x^2+y^2)-1}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases} \qquad f \in C^0(\mathbb{R}^2)$$

$$|f(\rho\cos\theta, \rho\sin\theta)| = \left| \rho(\cos\theta + \sin\theta) \, \frac{\cos\rho^2 - 1}{\rho^2} \right| \leq \left| \frac{\cos\rho^2-1}{\rho} \right| \xrightarrow[\rho \to 0]{} 0$$

$\cos\rho^2 = 1 - \frac{1}{2}\rho^2 + o(\rho^4) \Rightarrow f$ CONTINUA in $(0,0)$

Dv : $g(t) = f(t\cos\theta, t\sin\theta) = (\cos\theta + \sin\theta)\, \dfrac{\cos t^2 - 1}{t}$

Per θ fissato e $t \to 0$

$$g(t) \sim -\frac{1}{2} t (\cos\theta + \sin\theta) \Rightarrow g'(t) = -\frac{1}{2}(\cos\theta + \sin\theta)$$

$$\Rightarrow g'(0) = -\frac{1}{2}(\cos\theta + \sin\theta)$$

$$D_v f(0,0) = g'(0) = -\frac{1}{2}(\cos\theta + \sin\theta) \qquad \hat{v} = (\cos\theta, \sin\theta)$$

In particolare $\quad \dfrac{\partial f}{\partial x}(0,0) = -\dfrac{1}{2} \quad \theta = 0$

$$\frac{\partial f}{\partial y}(0,0) = -\frac{1}{2} \quad \theta = \pi/2$$

$$\nabla f(0,0) \cdot (\cos\theta, \sin\theta) = -\frac{1}{2}(\cos\theta + \sin\theta) = D_v f(0,0)$$

La formula del gradiente è verificata.