Differenziabilità di $f(x,y)$ in $(0,0)$

Devo dimostrare che

$$\lim_{(x,y)\to(0,0)} \frac{f(x,y)-f(0,0)-f_x(0,0)x-f_y(0,0)y}{\sqrt{x^2+y^2}} = 0 \quad (*)$$

$$\lim_{(x,y)\to(0,0)} \underbrace{(x+y)\left[\frac{\cos(x^2+y^2)-1+(x^2+y^2)/2}{(x^2+y^2)^{3/2}}\right]}_{g(x,y)}$$

Passiamo in coord. polari

$$g(\rho\cos\theta,\rho\operatorname{sen}\theta) = (\operatorname{sen}\theta+\cos\theta)\,\frac{\cos\rho^2-1+\rho^2/2}{\rho^2} \underset{\rho\to0}{=} (\operatorname{sen}\theta+\cos\theta)\,\frac{o(\rho^2)}{\rho^2}$$

$$\cos\rho^2 = 1-\frac{1}{2}\rho^2+o(\rho^2)$$

$$|g(\rho\cos\theta,\rho\operatorname{sen}\theta)| \le 2\,\frac{o(\rho^2)}{\rho^2} \xrightarrow[\rho\to0]{} 0$$ $\Rightarrow$ $(*)$ è verificato

$\Rightarrow$ f è differenziabile in $(0,0)$

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