5) $f(x,y) = e^{-x}(2x^2+y^2+4y)$ $\quad f \in C^\infty(\mathbb{R}^2)$

$$\begin{cases} \dfrac{\partial f}{\partial x} = (-2x^2-y^2-4y+4x)e^{-x} = 0 \\[2ex] \dfrac{\partial f}{\partial y} = (2y+4)e^{-x} = 0 \end{cases} \iff \begin{cases} -2x^2-y^2-4y+4x=0 \\[2ex] 2y+4=0 \end{cases}$$

$$\begin{cases} y=-2 \\ -2x^2-4+8+4x=0 \end{cases} \qquad \begin{cases} y=-2 \\ 2x^2-4x-4=0 \end{cases} \begin{cases} y=-2 \\ x^2-2x-2=0 \end{cases}$$

$$x = 1 \pm \sqrt{1+2} = 1 \pm \sqrt{3} \qquad P_1(1-\sqrt{3},-2) \quad P_2(1+\sqrt{3},-2)$$

pti critici

$$\frac{\partial^2 f}{\partial x^2} = e^{-x}(+2x^2+y^2+4y-4x-4x+4) = e^{-x}(2x^2+y^2+4y-8x+4)$$

$$\frac{\partial^2 f}{\partial y^2} = 2e^{-x} \qquad \frac{\partial^2 f}{\partial x \partial y} = -(2y+4)e^{-x}$$

$$H(1-\sqrt{3},-2) = \begin{pmatrix} 4\sqrt{3} & 0 \\ 0 & 2 \end{pmatrix} e^{1-\sqrt{3}}$$

La matrice hessiana è def >0 $\Rightarrow P_1$ è un minimo

$$H(1+\sqrt{3},-2) = \begin{pmatrix} -4\sqrt{3} & 0 \\ 0 & 2 \end{pmatrix} e^{1+\sqrt{3}}$$

indefinito $\Rightarrow P_2$ pto sella

$$z = f(1,1) + \frac{\partial f}{\partial x}(1,1)(x-1) + \frac{\partial f}{\partial y}(1,1)(y-1)$$ Piano tangente

$$z = 7e^{-1} + (-3e^{-1})(x-1) + 6e^{-1}(y-1)$$