6) $f(x,y) = 3y\, e^{-2(x-2)} + (x-1)y^2 + e^{x-2}$ $\qquad x_0 = 2$

$f \in C^\infty(\mathbb{R}^2)$

$3y + y^2 + 1 = 0 \qquad y^2 + 3y + 1 = 0$

$y = -3 \pm \sqrt{9-4} = -3 \pm \sqrt{5} \qquad y_1 = -3-\sqrt{5} \quad y_2 = -3+\sqrt{5}$

$\frac{\partial f}{\partial x} = 3y(-2)\, e^{-2(x-2)} + y^2 + e^{x-2}$ $\qquad f_y \neq 0$ e $f \in C^\infty \Rightarrow$

Posso applicare TEO DINI

$\frac{\partial f}{\partial y} = 3e^{-2(x-2)} + 2y(x-1) \qquad \left.\frac{\partial f}{\partial y}\right|_{(2,-3-\sqrt{5})} = 3 + 2(-3-\sqrt{5}) = -3-2\sqrt{5} \neq 0$

$$g'(2) = -\frac{3(-3-\sqrt{5})(-2) + (-3-\sqrt{5})^2 + 1}{-3-2\sqrt{5}} = +\frac{18+6\sqrt{5}+9+5+6\sqrt{5}+1}{3+2\sqrt{5}}$$

$$g'(2) = \frac{32+12\sqrt{5}}{3+2\sqrt{5}}$$